CBSE 10th Mathematics 2016 Solved Paper

All India

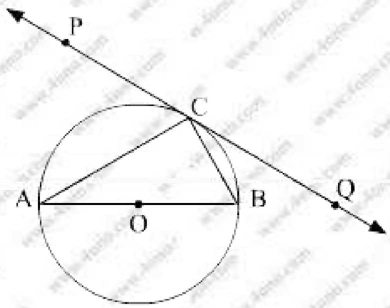
CBSE 10th Mathematics 2016 Solved Paper All India

TIME - 3HR. | QUESTIONS - 31

THE MARKS ARE MENTIONED ON EACH QUESTION

SECTION - A

Q.1. In fig. 1, PQ is a tangent at a point C to a circle with centre O. If AB is a diameter and ∠CAB = 30°, find ∠PCA. *1 mark*

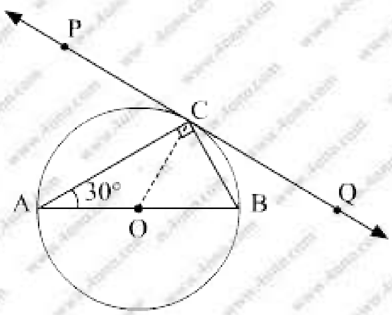


Solution:

Given:
∠CAB = 30°

AB is a diameter to the circle with centre O.
∴ ∠ACB = 90°

Join OC.



∴ OC = OA (Radii of the circle)

∠CAB = ∠ACO = 30°

We know that the tangent at any point of a circle is perpendicular to the radius through the point of contact.
∴ ∠OCP = 90°

$$\angle PCA + \angle OCA = 90°$$

$$\angle PCA = 90^0 - 30^0 = 60^0$$

$$\therefore \angle PCA = 60°.$$

Q.2. For what value of k will k + 9, 2k – 1 and 2k + 7 are the consecutive terms of an A.P.? *1 mark*

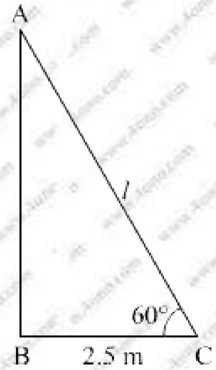
Solution:

If a, b and c are in AP, then $2b = a + c$.
It is given that $k + 9$, $2k - 1$ and $2k + 7$ are in AP.
$\therefore 2(2k - 1) = (k + 9) + (2k + 7)$
$\Rightarrow 4k - 2 = 3k + 16$
$\Rightarrow k = 18$
Thus, the value of k is 18.

Q.3. A ladder, leaning against a wall, makes an angle of 60° with the horizontal. If the foot of the ladder is 2.5 m away from the wall, find the length of the ladder. .? *1 mark*

Solution:

The given information is represented in the figure shown below:



Here, AC is the ladder with length l and AB is the wall.
In ΔABC,

$$\cos 60^0 = \frac{BC}{AC} \quad \left(\because \cos\theta = \frac{Base}{Hypotenuse}\right)$$

$$\Rightarrow \frac{1}{2} = \frac{2.5}{l}$$

$$\Rightarrow l = 2.5 \text{ x } 2 = 5 \text{ metres}$$

Thus, the length of the ladder is 5 metres.

Q.4 A card is drawn at random from a well shuffled pack of 52 playing cards. Find the probability of getting neither a red card nor a queen. *1 mark*

Solution:

Out of 52 cards, one card can be drawn in 52 ways.
∴ Total number of events = 52
In a pack of 52 playing cards, there are 26 red cards and 26 black cards that include 2 red queens and 2 black queens, respectively.
∴ Number of cards that are neither red nor queen = 52 − (26 + 2) = 24
⇒ Favourable number of events = 24
∴ Required probability $= \frac{24}{52} = \frac{6}{13}$.

SECTION - B

Q.5. If –5 is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots, find the value of k. *2 marks*

Solution:

-5 *is a root of the quadratic equation* $2x^2 + px - 15 = 0$.
$\therefore\ 2(-5)^2 + p(-5) - 15 = 0$
$\Rightarrow 50 - 5p - 15 = 0$
$\Rightarrow 35 = 5p$
$\Rightarrow p = 7$
It is given that $p(x^2 + x) + k = 0$.
$7(x^2 + x) + k = 0$
$\Rightarrow 7x^2 + 7x + k = 0$
If the roots are equal then, $D = 0$.
$D = b^2 - 4ac = 0$
$\Rightarrow 49 - 28k = 0$
$\Rightarrow 49 = 28k$
$\Rightarrow k = \frac{7}{4}$.

Q.6. Let P and Q be the points of trisection of the line segment joining the points A(2, –2) and B(–7, 4) such that P is nearer to A. Find the coordinates of P and Q. 2 marks

Solution:

It is given that P and Q are the points of trisection of the line segment joining points A(2, –2) and B(–7, 4) such that P is nearer to A. Therefore, P divides the line segment AB internally in the ratio 1 : 2 and Q divides AB internally in the ratio 2 : 1.

Using section formula, we have

Coordinates of P

$$= \left(\frac{1 \times (-7) + 2 \times 2}{1+2}, \frac{1 \times 4 + 2 \times (-2)}{1}\right)$$

$$= \left(\frac{-7+4}{3}, \frac{4-4}{3}\right)$$

$$= (-1, 0)$$

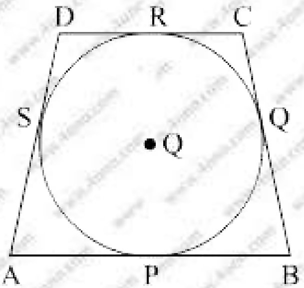
Coordinates of Q

$$= \left(\frac{2 \times (-7) + 1 \times 2}{2+1}, \frac{2 \times 4 + 1 \times (-2)}{2+1}\right)$$

$$= \left(\frac{-14+2}{3}, \frac{8-2}{3}\right)$$

$$= (-4, 2).$$

Q.7. In Figure, a quadrilateral ABCD is drawn to circumscribe a circle with centre O in such a way that the sides AB, BC, CD and DA touch the circle at the points P, Q, R and S respectively. Prove that AB+ CD = BC + DA. 2 marks



Solution:

We know that the tangents drawn from the exterior point to a circle are equal in length.
So,
From point D, DR = DS .....(i)
From point A, AP = AS .....(ii)
From point B, BP = BQ .....(iii)
From point C, CR = CQ .....(iv)

Adding (i), (ii), (iii) and (iv), we get
DR + AP + BP + CR = DS + AS + BQ + CQ
(DR + CR) + (AP + BP) = (DS + AS) + (BQ + CQ)
CD + AB = DA + BC
AB + CD = BC + DA
Hence proved.

Q.8. Prove that the points (3, 0), (6, 4) and (−1, 3) are the vertices of a right angled isosceles triangle. *2 marks*

Solution:

Let A(3, 0), B(6, 4) and C(−1, 3) be the vertices of the given triangle.
Using distance formula, we have

$$AB = \sqrt{(4-0)^2 + (6-3)^2} = \sqrt{25} = 5\ units$$

$$AB = \sqrt{(3-4)^2 + (-1-6)^2} = \sqrt{50} = 5\sqrt{2}\ units$$

$$CA = \sqrt{(0-3)^2 + (3+1)^2} = \sqrt{25} = 5\ units$$

Now,

$$(5)^2 + (5)^2 = (5\sqrt{2})^2$$

$$\Rightarrow AB^2 + CA^2 = BC^2$$

⇒ ΔABC is a right-angled triangle, right angled at A.
Also,
AB = CA = 5 units
Therefore, ΔABC is a right-angled isosceles triangle.
Hence, the given points are the vertices of a right-angled isosceles triangle.

Q.9. The 4th term of an A.P. is zero. Prove that the 25th term of the A.P. is three times its 11th term. *2 marks*

Solution:

Let a and d be the first term and the common difference of the AP, respectively.
It is given that $a_4 = 0$.
$\therefore a + 3d = 0$
$\Rightarrow a = -3d$
Now,
$a_{25} = a + 24d$

$$\Rightarrow a_{25} = -3d + 24d = 21d \quad .....(1)$$

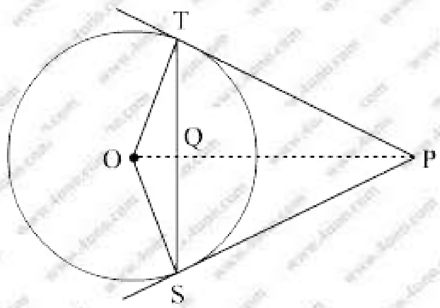
Also,
$a_{11} = a + 10d$

$$\Rightarrow a_{11} = -3d + 10d = 7d \quad .....(2)$$

From (1) and (2), we have

$$a_{25} = 3 \times a_{11}.$$

Q.10. In Fig. 3, from an external point P, two tangents PT and PS are drawn to a circle with centre O and radius *r*. 2 marks
If OP = 2*r*, show that ∠OTS = ∠OST = 30°.



Solution:

Given:
O is the centre and r is the radius of the circle.
PT and PS are tangents to the circle.
OP = 2r

To prove: $\angle OTS = \angle OST = 30^0$

Proof:
PT and PS are tangents drawn to the circle.

$\therefore \angle OTP = \angle OSP = 90^0$ (Tangent to a circle is perpendicular to the radius through the point of contact.)

In ΔOTP,

$$sin\angle OPT = \frac{OT}{OP} = \frac{r}{2r}$$

$$\Rightarrow sin\angle OPT = \frac{1}{2}$$

$$\Rightarrow \angle OPT = 30^0$$

Now,

$\angle OTP + \angle OPT + \angle TOP = 180^0$ $(Angle\ sum\ property)$

$\Rightarrow 90^0 + 30^0 + \angle TOP = 180^0$

$\Rightarrow \angle TOP = 180^0 - 120^0 = 60^0$

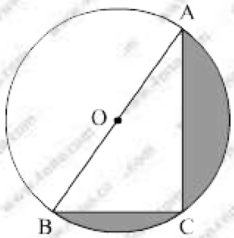
ΔPTS is an isosceles triangle and OP is the angle bisector of ∠TPS

$\Rightarrow TS \perp OP$

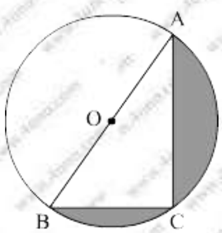
$\therefore \angle OQT = \angle OQS = 90°$
$In\ \Delta OTQ,$
$\angle OQT + \angle OTQ + \angle TOQ = 180°$ $(Angle\ sum\ property)$
$\Rightarrow 90° + \angle OTQ + 60° = 180° \Rightarrow \angle OTQ = 180° - 150° = 30°$
Similarly,
$\angle OSQ = 30°$
$\therefore \angle OTS = \angle OST = 30°.$

SECTION - C

Q.11. In fig. 4, O is the centre of a circle such that diameter AB = 13 cm and AC = 12 cm. BC is joined. Find the area of the shaded region. (Take π = 3.14) *3 marks*



Solution:



It is given that AB = 13 cm and AC = 12 cm.
We know that angle inscribed in a semicircle is 90°.

∴ ∠ACB = 90°
So, ΔABC is a right-angled triangle.
In ΔABC,
$AC^2 + BC^2 = AB^2$
⇒ 144 + BC^2 = = 169
⇒ BC^2 = = 25
⇒ BC = 5 cm

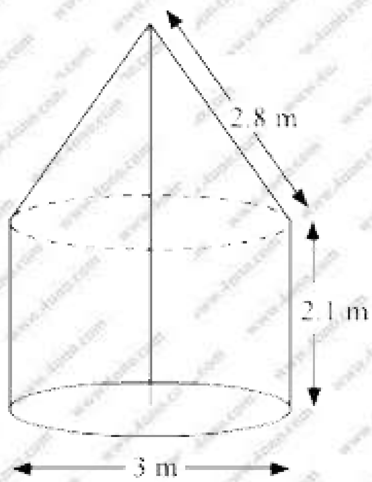
$$Area\ of\ \Delta ABC = \frac{1}{2} \times BC \times AC = \frac{1}{2} \times 5 \times 12 = 30\ cm^2$$

Radius of the circle, $r = \frac{AB}{2} = \frac{13}{2}\ cm$

Area of semicircle $ACB = \frac{\pi r^2}{2} = \frac{3.14}{2} \times \frac{13}{2} \times \frac{13}{2} = 66.33\ cm^2$ (Approx.)

∴ Area of the shaded region = Area of semicircle ACB − Area of ΔACB = 66.33 − 30 = 36.33 cm^2 (Approx.)

Q.12. In the figure, a tent is in the shape of a cylinder surmounted by a conical top of same diameter. If the height and diameter of cylindrical part are 2.1 m and 3 m, respectively, and the slant height of conical part is 2.8 m, find the cost of canvas needed to make the tent if the canvas is available at the rate of Rs 500/sq. metre. $\left(Use\ \pi = \frac{22}{7}\right)$. 3 marks



Solution:

Canvas needed to make the tent = Curved surface area of the conical part + Curved surface area of the cylindrical part

Radius of the conical part = Radius of the cylindrical part $r = \frac{3}{2} m$

Slant height of the conical part = l = 2.8 m
Height of the cylindrical part = h = 2.1 m

Curved surface area of the conical part $= \pi r l = \frac{22}{7} \times \frac{3}{2} \times\ 2.8\ m^2$

Curved surface area of the cylindrical part $= \ 2\pi r h = 2 \times \frac{22}{7} \times \frac{3}{2} \times 2.1\ m^2$

∴ Total area of the canvas needed to make the tent

$= \frac{22}{7} \times \frac{3}{2} \times 2.8 + 2 \times \frac{22}{7} \times \frac{3}{2} \times 2.1$

$= \frac{22}{7} \times \frac{3}{2} \times (2.8 + 4.2\ )$

$= \frac{22}{7} \times \frac{3}{2} \times 7$

$= 33\ m^2.$

Cost of the canvas $=$ ₹$500/m^2$

∴ Total cost of the canvas needed to make the tent = 500 × 33 = ₹16,500.

Q.13. If the point P(x, y) is equidistant from the points A(a + b, b − a) and B(a − b, a + b). Prove that bx = ay. 3 marks

Solution:

It is given that the point P(x, y) is equidistant from the points A($a + b$, $b - a$) and $B(a - b,\ a + b)$.

$\therefore\ PA = PB$

$\Rightarrow \sqrt{(a+b-x^2+(b-a-y^2)} = \sqrt{(a-b-x^2)+(a+b-y^2)}$

$\Rightarrow (a+b-x^2)+(b-a-x^2) = (a-b-x^2)+(a+b-y^2)$

$\Rightarrow (a+b-x^2)-(a-b-x^2) = (a+b-y^2)-(b-a-y^2)$

$\Rightarrow (a+b-x+a-b-x)(a+b-x-a+b+x)$
$= (a+b-y+b-a-y)(a+b-y-b+a+y)$

$\Rightarrow (2a-2x)(2b) = (2b-2y)(2a)$

$\Rightarrow (a-x)b = (b-y)a$

$\Rightarrow ab - bx = ab - ay \Rightarrow bx = ay.$