CBSE 12th Mathematics 2016 Solved Paper Outside Delhi

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TIME - 3HR. | QUESTIONS - 26

THE MARKS ARE MENTIONED ON EACH QUESTION

Question numbers 1 to 6 carry 1 mark each.

SECTION - A

Q.1. If $x \in N$ and $\begin{vmatrix} x+3 & -2 \\ -3x & 2x \end{vmatrix} = 8$, then find the value of x. *1 mark*

Ans. $\begin{vmatrix} x+3 & -2 \\ -3x & 2x \end{vmatrix} = 8$

$\Rightarrow 2x^2 + 6x - 6x = 8$

$\Rightarrow 2x^2 = 8$

$\Rightarrow x^2 = 4$

$\Rightarrow x = 2$

$\therefore x \in \mathbb{N}$.

Q.2. Use elementary column operation $C_2 \to C_2 + 2C_1$ in the following matrix equation:

$\begin{pmatrix} 2 & 1 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$. *1 mark*

Ans. $\begin{pmatrix} 2 & 1 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$

Using $C_2 \to C_2 + 2C_1$

$\begin{pmatrix} 2 & 5 \\ 2 & 4 \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ -1 & -1 \end{pmatrix}$.

Q.3. Write the number of all possible matrices of order 2×2 with each entry 1, 2 or 3. *1 mark*

Ans. $3^4 = 81$

Q.4. Write the position vector of the point which divides the join of points with position vectors $3\vec{a} - 2\vec{b}$ and $2\vec{a} + 3\vec{b}$ in the ratio 2 : 1. *1 mark*

Ans. Let $\overrightarrow{OP}$ be the required vector $i.e.$

$$\overrightarrow{OP} = \frac{2(2\vec{a} + 3\vec{b}) + 1(3\vec{a} - 2\vec{b})}{2+1}$$
$$= \frac{7\vec{a} + 4\vec{b}}{3}.$$

Q.5. Write the number of vectors of unit length perpendicular to both the vectors of unit length perpendicular to both the vectors $\vec{a} = 2\hat{\imath} + \hat{\jmath} + 2\hat{k}$ and $\vec{b} = \hat{\jmath} + \hat{k}$. *1 mark*

Ans.2.

Q.6. Find the vector equation of the plane with intercepts 3, −4 and 2 on x, y and $z -$ axis respectively. *1 mark*

Ans.

$$\frac{x}{3} + \frac{y}{-4} + \frac{z}{2} = 1$$
$$\Rightarrow \frac{x}{3} - \frac{y}{4} + \frac{z}{2} = 1 \text{ be the eq. of plane.}$$

SECTION – B

Question numbers 7 to 19 carry 4 marks each.

Q.7. Find the coordinates of the point where the line through the points A (3, 4, 1) and B (5, 1, 6) crosses the XZ plane. Also find the angle which this line makes with the XZ plane. *4 marks*

Ans. Eq. of line through A (3, 4, 1) and B (5, 1, 6) be:

$$\frac{x-3}{5-3} = \frac{y-4}{1-4} = \frac{z-1}{6-1}$$
$$\Rightarrow \frac{x-3}{2} = \frac{y-4}{-3} = \frac{z-1}{5}$$

Let the point of intersection of line and xz plane be (x_0, y_0, z_0) $i.e.$ it lie on line

$$\frac{x_0-3}{2} = \frac{y_0-4}{-3} = \frac{z_0-1}{5} = \lambda$$
$$x_0 = 2\lambda + 3,$$
$$y_0 = -3\lambda + 4, z_0 = 5\lambda + 1$$

It also lie on xz plane so,

$$y_0 = 0$$
$$\Rightarrow -3\lambda + 4 = 0$$
$$\Rightarrow \lambda = 4/3$$
$$i.e.\ \ x_0 = 2(4/3) + 3 \quad \& z_0 = 5(4/3) + 1$$

$$= \frac{8+9}{3} \qquad = \frac{20+3}{3}$$

$$= \frac{17}{3} \qquad = \frac{23}{3}$$

$$i.e., \qquad Pt.\text{ be } \left(\frac{17}{3}, 0, \frac{23}{3}\right)$$

Direct of line AB is $(2, -3, 5)$ and Direction of plane xz is $(0, 1, 0)$

Let angle between line and plane is θ $i.e.,$ angle is $\sin\theta$

$$= \left(\frac{2(0) + (-3)1 + 5(0)}{\sqrt{2^2 + (-3)^2 + (5)^2}}\right)$$

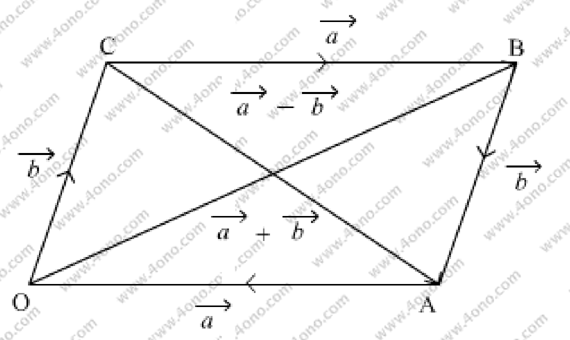
$$= \left(\frac{3+0}{\sqrt{38}}\right) = \left(\frac{3}{\sqrt{38}}\right)$$

$$\theta = \sin^{-1}\left(\frac{3}{\sqrt{38}}\right)$$

Q.8. The two adjacent sides of parallelogram are $2\hat{\imath} - 4\hat{\jmath} - 5\hat{k}$ and $2\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$. Find the two unit vectors parallel to its diagonals. Using the diagonal vectors, find the area of the parallelogram. *4 marks*

Ans. Let OABC be a parallelogram with side $\overrightarrow{OA} = \vec{a} = 2\hat{\imath} - 4\hat{\jmath} - 5\hat{k}$ and

$$\overrightarrow{AB} = \vec{b} = 2\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$$



Now diagonal $\overrightarrow{OB} = \vec{a} + \vec{b} = \overrightarrow{OA} + \overrightarrow{AB}$

$$= 4\hat{\imath} - 2\hat{\jmath} - 2\hat{k}$$

diagonal $\overrightarrow{CA} = \overrightarrow{CB} - \overrightarrow{BA} = \vec{a} - \vec{b}$

$$= 0\hat{\imath} - 6\hat{\jmath} - 8\hat{k}$$

$$\widehat{OB} = \frac{\overrightarrow{OB}}{|\overrightarrow{OB}|} = \frac{4\hat{\imath} - 2\hat{\jmath} - 2\hat{k}}{\sqrt{16+4+4}}$$

$$= \frac{4}{\sqrt{24}}\hat{\imath} - \frac{2}{\sqrt{24}}\hat{\jmath} - \frac{2}{\sqrt{24}}\hat{k}$$

$$= \frac{2}{\sqrt{6}}\hat{\imath} - \frac{1}{\sqrt{6}}\hat{\jmath} - \frac{1}{\sqrt{6}}\hat{k}$$

$$\widehat{CA} = \frac{-6\hat{\jmath} - 8\hat{k}}{\sqrt{0^2 + 64 + 36}}$$
$$= \frac{-6}{10}\hat{\jmath} - \frac{8}{10}\hat{k} = \frac{-3}{5}\hat{\jmath} - \frac{4}{5}\hat{k}$$

$i.e.$, unit vector along diagonal be $\widehat{OB}$ and $\widehat{CA}$.

Now area of parallelogram be

$$= \frac{1}{2}\left|\overrightarrow{OB}\times\overrightarrow{CA}\right|$$

$$\overrightarrow{OB}\times\overrightarrow{CA} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 4 & -2 & -2 \\ 0 & -6 & -8 \end{vmatrix}$$
$$= \hat{\imath}(16 - 12) - \hat{\jmath}(-32) + \hat{k}(-24)$$
$$= 4\hat{\imath} + 32\hat{\jmath} - 24\hat{k}$$

$$\left|\overrightarrow{OB}\times\overrightarrow{CA}\right| = \sqrt{4^2 + (32)^2 + (24)^2}$$
$$= \sqrt{16 + 1024 + 576}$$
$$= \sqrt{1616} = 4\sqrt{101}$$

Area of parallelogram be = $\frac{1}{2}(4\sqrt{101})$

$$= 2\sqrt{101}\ sq.unit.$$

Q.9. In a game, a man wins Rs5 for getting a number greater than 4 and loses Rs1 otherwise, when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a number greater than 4. Find the expected value of the amount he win/lose. 4 marks

Ans. Let x denote the amount he win/loss $i.e.$, $x = 5, 4, 3, -3$ win in first thrown

$P(x = 5) =$ win in first thrown 2/6 = 1/3 = 9/27
$P(x = 4) =$ win in second thrown

$$= \frac{4}{6}\times\frac{2}{6} = \frac{2}{9} = \frac{6}{27}$$

$P(x = 3) =$ win in third thrown

$$= \left(\frac{4}{6}\right)\times\frac{4}{6}\times\frac{2}{6} = \frac{4}{27}$$

$P(x = -3) =$ not win in or third thrown

$= \left(\frac{4}{6}\right)^3 = \frac{8}{27}$

$$E(x) = \Sigma x P(x) = 5.\frac{9}{27} + 4.\frac{6}{27} + 3.\frac{4}{27} - 3.\frac{8}{27}$$
$$= \frac{45 + 24 + 12 - 24}{27} = \frac{57}{27}$$

OR

A bag contains 4 balls. Two balls are drawn at random (without replacement) and are found to be white. What is the probability that all balls in the bag are white?

Ans. Let E_1 be event the bag has 4 white balls.
E_2 be event the bag has no white balls
E_3 be event the bag has 3 white balls
E_4 be event to draw 2 balls from balls

A be event to draw 2 balls from balls and are white

$P(E_1) = 1/4$
$P(E_2) = 1/4$
$P(E_3) = 1/4$
$P(E_4) = 1/4$

$P(A/E_1) = 1$
$P(A/E_2) = 0$
$P(A/E_3) = \frac{3}{4} \times \frac{1}{3} = \frac{1}{6}$

$$= \frac{\frac{1}{4} \times 1}{\left(\frac{1}{4} \times 1\right)\left(\frac{1}{4} \times 0\right)\left(\frac{1}{4} \times \frac{1}{2}\right)\left(\frac{1}{4} \times \frac{1}{6}\right)}$$

$$= \frac{1}{1 + \frac{1}{2} + \frac{1}{6}} = \frac{1}{\frac{6+3+1}{6}} = \frac{6}{10} = \frac{3}{5}.$$

Q.10. Differentiate $x^{\sin x} + (\sin x)^{\cos x}$ with respect to x. 4 marks

Ans. *Let* $y = x^{\sin x} + (\sin x)^{\cos x}$ and $u = x^{\sin x}$

taking both side

$$\log u = \log x^{\sin x}$$
$$\log u = \log x \log x$$

Differentiate w.r.t. x

$$\frac{1}{u}\frac{du}{dx} = \left(\cos x \log x + \frac{\sin x}{x}\right)$$

$$\frac{du}{dx} = u\left[\frac{x\cos x\log x + \sin x}{x}\right]$$

$$= x^{\sin x}\left[\frac{x\cos x\log x + \sin x}{x}\right]$$

$$= x^{(\sin x - 1)}[x\cos x\log x + \sin x]$$

$$v = (\sin x)^{\cos x}$$

Taking log both side

$$\log v = \log(\sin x)^{\cos x}$$
$$\log v = \cos x \log \sin x$$

Differentiate w.r.t. x

$$\frac{1}{v}\frac{dv}{dx} = -\sin x\log(\sin x) + \cos x\,(\cot x)$$
$$= (\sin x)^{\cos x}\left[(-\sin x\log^{\sin x}) + \cos x\cot x\right]$$

Now

$$y = u + v$$

⇒ differentiate w.r.t. x

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$= x^{(\sin x - 1)}[x\cos x\log x + \sin x] + (\sin x)^{\cos x}[-\sin x\log(\sin x) + \cos x\cot x].$$

OR

If $y = 2\cos(\log 3) + 3\sin(\log x)$, prove that

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0.$$

Ans. $y = 2\cos(\log x) + 3\sin(\log x)$ differentiate w.r.t. x

$$\frac{dy}{dx} = \frac{-2\sin(\log x)}{x} + \frac{3\cos(\log x)}{x}$$

$$x\frac{dy}{dx} = -2\sin(\log x) + 3\cos(\log x)$$

Again differentiate w.r.t. x

$$(1)\frac{dy}{dx} + x\frac{d^2y}{dx^2} = \frac{-2\cos(\log x)}{x} + 3\left[\frac{-\sin(\log x)}{x}\right]$$

$$\Rightarrow x\frac{d^2y}{dx^2}+\frac{dy}{dx}=-\left(\frac{2\cos(\log x)+3\sin(\log x)}{x}\right)$$

$$\Rightarrow x^2\frac{d^2y}{dx^2}+\frac{dy}{dx}=-y$$

$$\Rightarrow x^2\frac{d^2y}{dx^2}+\frac{dy}{dx}+y=0.$$

Q.11. If $x=a\sin 2t\,(1+\cos 2t)$ and $y=b\cos 2t\,(1-\cos 2t)$, find $\frac{dy}{dx}$ at $t=\frac{\pi}{4}$. 4 marks

Ans. $x=a\sin 2t\ (1+\cos 2t)$ and $y=b\cos 2t\ (1-\cos 2t)$

differentiate w.r.t. x

$$\frac{dx}{dt}=a[2\cos 2t\,(1+\cos 2t)+\sin 2t\,(-2\sin 2t)]$$
$$=2a[\cos 2t+\cos^2 2t-\sin^2 2t]$$
$$=2a[\cos 2t+\cos 4t]$$

$$\frac{dy}{dt}=b[2(-\sin 2t)(1-\cos 2t)+\cos 2t\,(\sin 2t)2]$$
$$=2b[-\sin 2t+\sin 2t\cos 2t+\sin 2t\cos 2t]$$
$$=2b\,[-\sin 2t+\sin 4t]$$

Now,

$$\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{2b(\sin 4t-\sin 2t)}{2a(\cos 2t+\cos 4t)}$$
$$=\frac{b}{a}\left(\frac{\sin 4t-\sin 2t}{\cos 2t+\cos 4t}\right)$$
$$\left.\frac{dy}{dx}\right]_{t=\pi/4}=\frac{b}{a}\left(\frac{0-1}{-1+0}\right)=\frac{b}{a}.$$

Q.12. The equation to tangent at (2, 3) on the curve $y^2=ax^3+b$ is $y=4x-5$. Find the values of a and b. 4 marks

Ans. $y^2=ax^3+b$ and pt. is (2, 3)

Differentiate w.r.t. x

$$2y\frac{dy}{dx}=3ax^2$$

$$\frac{dy}{dx}=\frac{3ax^2}{2y}$$

$$m = \frac{dy}{dx}\bigg]_{(2,3)} = \frac{12a}{6}$$

$= 2a =$ Slope of tangent.

and eq. of tangent is

$$y - 3 = m(x - 2)$$
$$y - 3 = 2a(x - 2)$$
$$y = 2ax - 4a + 3$$

Now compare with $y = 4x - 5$ $i.e.,$

$2a = 4$ and $-4a + 3 - 5$

$$a = 4 \quad -4a = -8$$

$$a = 2$$

Pt. (2, 3) also lie on curve $i.e.,$

$$9 = 8a + b$$
$$9 = 8(2) + b$$
$$b = 9 - 16 = -7$$

$i.e.,$ $a = 2$ and $b = -7$.

Q.13. Find: *4 marks*

$$\int \frac{x^2}{x^4 + x^2 - 2} dx$$

Ans.

$$I = \int \frac{x^2}{x^4 + x^2 - 2} dx = \int \frac{x^2 dx}{x^4 + 2x^2 - x^2 - 2}$$

$$\int \frac{x^2 dx}{(x^2 + 2)(x^2 - 1)}$$

Now let

$$\frac{x^2 dx}{(x^2 + 2)(x^2 - 1)}$$

$$= \frac{A}{(x^2 + 2)} + \frac{B}{(x^2 - 1)}$$

$$x^2 = A(x^2 - 1) + B(x^2 + 2)$$
$$\Rightarrow x^2 = (A + B)x^2 + (A + 2B)$$

Comparing coefficient of x^2 and constant $i.e.$,

$A + B = 1$ and $-A + 2B = 0$

$\Rightarrow 3B = 1$
$B = 1/3$

and $A = 1 - 1/3 = 2/3$

Now,

$$I = \int \frac{2/3}{(x^2+2)} + \frac{1/3}{(x^2-1)} dx$$

$$= \frac{2}{3}\int \frac{dx}{x^2 + (\sqrt{2})^2} + \frac{1}{3}\int \frac{dx}{x^2-1}$$

$$= \frac{2}{3}\left[\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right] + \frac{1}{3}\left[\frac{1}{2}\log\left(\frac{x-1}{x+1}\right)\right] + C$$

$$= \frac{2}{3\sqrt{3}}\tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{6}\log\left(\frac{x-1}{x+1}\right) + C.$$

Q.14. Evaluate: *4 marks*

$$\int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin x + \cos x} dx$$

Ans.

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin x + \cos x} dx \qquad \ldots(i)$$

$$\left\{Apply \int_0^a f(x)dx = \int_0^a f(a-x)dx\right\}$$

Now,

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin^2(x/2 - x)}{\sin(x/2 - x) + \cos(x/2 - x)} dx$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{\cos x + \sin x} dx \qquad \ldots(ii)$$