ICSE Paper 2013

MATHEMATICS

SECTION A [40 Marks]

*(Answer **all** questions from this Section.)*

Question 1.

(a) Given $A = \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix}, B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}, C = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}.$

Find the matrix X such that $A + 2X = 2B + C.$ **[3]**

(b) At what rate % p.a. will a sum of ₹ 4000 yield ₹ 1324 as compound interest in 3 years ? **[3]**

(c) The median of the following observations 11, 12, 14, (x – 2), (x + 4), (x + 9), 32, 38, 47 arranged in ascending order is 24. Find the value of x and hence find the mean. **[4]**

Solution :

(a) *Given :* $A = \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix}, B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$

$\therefore \qquad A + 2X = 2B + C$

Putting the given values, we get

$$\begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix} + 2X = 2\begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$$

$$2X = \begin{bmatrix} -6+4 & 4+0 \\ 8+0 & 0+2 \end{bmatrix} - \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix}$$

$$X = \frac{1}{2}\begin{bmatrix} -4 & 10 \\ 6 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} -2 & 5 \\ 3 & 1 \end{bmatrix} \qquad \textbf{Ans.}$$

(b) *Given :* Principal = ₹ 4,000, C.I. = ₹ 1,324,

Amount = P + C.I.

$= ₹ (4,000 + 1,324) = ₹ 5,324$

Time = 3 years

We know that, $A = P\left(1 + \dfrac{r}{100}\right)^T$

$$5,324 = 4,000\left(1 + \frac{r}{100}\right)^3$$

$$\frac{5,324}{4,000} = \left(1 + \frac{r}{100}\right)^3$$

$$\frac{1,331}{1,000} = \left(1 + \frac{r}{100}\right)^3$$

$$\left(\frac{11}{10}\right)^3 = \left(1 + \frac{r}{100}\right)^3$$

Therefore,
$$1 + \frac{r}{100} = \frac{11}{10}$$

$$\frac{r}{100} = \frac{11}{10} - 1$$

$$\frac{r}{100} = \frac{1}{10}$$

$$r = \frac{100}{10}$$

$$r = 10\% \qquad \text{Ans.}$$

(c) Given observation are 11, 12, 14, $(x-2)$, $(x+4)$, $(x+9)$, 32, 38, 47 and median = 24.

$$n = 9 \text{ (odd)}$$

$$\therefore \quad \text{Median} = \frac{n+1}{2} \text{ th term}$$

$$= \frac{9+1}{2} \text{ th term}$$

$$24 = 5\text{th term}$$

$$\therefore \quad x + 4 = 24$$

$$x = 24 - 4$$

$$x = 20$$

Therefore, 11, 12, 14, $(20-2)$, $(20+4)$, $(20+9)$, 32, 38, 47

$$= 11, 12, 14, 18, 24, 29, 32, 38, 47$$

Now
$$\text{Mean} = \frac{\Sigma x}{n}$$

$$= \frac{11 + 12 + 14 + 18 + 24 + 29 + 32 + 38 + 47}{9}$$

$$= \frac{225}{9} = 25. \qquad \text{Ans.}$$

Question 2.

(a) What number must be added to each of the number 6, 15, 20 and 43 to make them proportional ? [3]

(b) If $(x - 2)$ is a factor of the expression $2x^3 + ax^2 + bx - 14$ and when the expression is divided by $(x - 3)$, it leaves a remainder 52, find the values of a and b. [3]

(c) Draw a histogram from the following frequency distribution and find the mode from the graph : [4]

Class	0–5	5–10	10–15	15–20	20–25	25–30
Frequency	2	5	18	14	8	5

Solution :

(a) Let the number must be added be x, then

the new number $= 6 + x, 15 + x, 20 + x, 43 + x$

$\because$ These are proportionals.

$$6 + x : 15 + x :: 20 + x : 43 + x$$

or $\qquad (6 + x)(43 + x) = (15 + x)(20 + x)$

or $\qquad 258 + 6x + 43x + x^2 = 300 + 20x + 15x + x^2$

or $\qquad 49x - 35x = 300 - 258$

or $\qquad 14x = 42$

or $\qquad x = 3.$ **Ans.**

(b) Let $(x - 2)$ is a factor of the given expression.

$\because \qquad x - 2 = 0$

$x = 2$

Given expression,

$$2x^3 + ax^2 + bx - 14 = 0$$

$$2(2)^3 + a(2)^2 + b(2) - 14 = 0$$

$$16 + 4a + 2b - 14 = 0$$

$$4a + 2b + 2 = 0$$

$$4a + 2b = -2$$

$$2a + b = -1 \qquad \text{...(i)}$$

and when given expression is divided by $(x - 3)$

$$x - 3 = 0$$

$\Rightarrow \qquad x = 3$

$\therefore \qquad 2x^3 + ax^2 + bx - 14 = 52$

$$2(3)^3 + a(3)^2 + b(3) - 66 = 0$$

$$54 + 9a + 3b - 66 = 0$$

$$9a + 3b = 12$$

$$3a + b = 4 \qquad \text{...(ii)}$$

Solving equation (i) and (ii),

$$2a + b = -1$$

$$3a + b = 4$$

$$\underline{(-) \quad (-) \quad (+)}$$
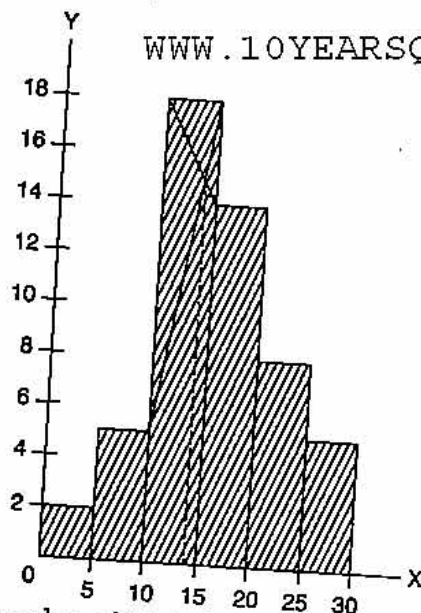
$$-a = -5$$

$$a = 5$$

from (ii),

$$3 \times 5 + b = 4$$

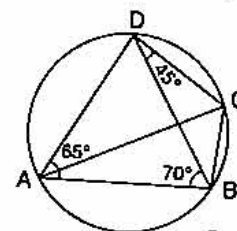$$b = 4 - 15$$

$$b = -11$$

$$a = 5 \text{ and } b = -11 \qquad \textbf{Ans.}$$

$\therefore$

(c)

From the Histogram the value of Mode is **13·8.**

Question 3.

Ans.

(a) *Without using tables evaluate 3 cos 80°.Cosec 10° + 2 sin 59° sec 31°.* [3]

(b) *In the given figure,*

$$\angle BAD = 65°,$$
$$\angle ABD = 70°,$$
$$\angle BDC = 45°$$

(i) *Prove that AC is a diameter of the circle.*

(ii) *Find ∠ACB.* [3]

(c) *AB is a diameter of a circle with centre C = (–2, 5). If A = (3, –7). Find :*

(i) *The length of radius AC*

(ii) *The coordinates of B.* [4]

Solution :

(a) *Given :*

$$3 \cos 80°·\text{cosec } 10° + 2 \sin 59° \sec 31°$$

$$= 3 \cos 80° \text{ cosec } (90° - 80°) + 2 \sin 59° \sec (90° - 59°)$$

$$= 3 \cos 80° \sec 80° + 2 \sin 59° \text{ cosec } 59°$$

$$= 3 \cos 80° \times \frac{1}{\cos 80°} + 2 \sin 59° \times \frac{1}{\sin 59°}$$

$$= 3 + 2 = 5.$$

(b) *Given :* ∠ BAD = 65°, ∠ ABD = 70°, ∠ BDC = 45° **Ans.**

(i) ∵ ABCD is a cyclic quadrilateral.

In Δ ABD,

∴ ∠ BDA + ∠ DAB + ∠ ABD = 180°   *By using sum property of Δˢ*

∠ BDA = 180° – (65° + 70°)

= 180° – 135°

= 45°

Now from $\triangle$ ACD,

$$\angle ADC = \angle ADB + \angle BDC$$
$$= 45° + 45° \qquad (\because \angle BDA = \angle ADB = 45°)$$
$$= 90°$$

Hence, $\angle D$ makes right angle belongs in semi-circle therefore AC is a diameter of the circle.

(ii) $\angle ACB = \angle ADB$ (Angles in the same segment of a circle)

$\therefore$ $\angle ACB = 45°$ $(\because \angle ADB = 45°)$ **Ans.**

(c)   (i)   The length of radius AC $= \sqrt{(-2-3)^2 + (5+7)^2}$

$$= \sqrt{(-5)^2 + (12)^2}$$
$$= \sqrt{25 + 144}$$
$$= \sqrt{169}$$
$$= 13. \qquad \textbf{Ans.}$$
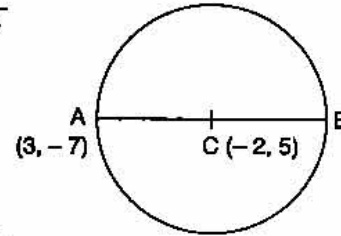
(ii)   Let the point of B be (x, y).

Given C is the mid-point of AB. Therefore

$$-2 = \frac{3+x}{2}$$
$$\Rightarrow \qquad 3 + x = -4$$
$$\Rightarrow \qquad x = -4 - 3 = -7$$
$$\text{and} \qquad 5 = \frac{-7+y}{2}$$
$$\Rightarrow \qquad 10 = -7 + y$$
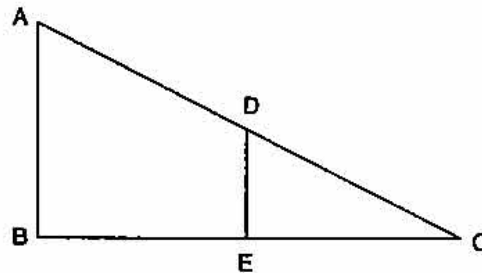$$y = 17$$

Hence, the co-ordinate of B $(-7, 17)$.   **Ans.**

Question 4.

(a) *Solve the following equation and calculate the answer correct to two decimal places :*

$$x^2 - 5x - 10 = 0. \qquad \text{[3]}$$

(b) *In the given figure, AB and DE are perpendicular to BC.*

   (i)   *Prove that $\triangle ABC \sim \triangle DEC$*

   (ii)   *If AB = 6 cm, DE = 4 cm and AC = 15 cm. Calculate CD.*

   (iii)   *Find the ratio of the area of $\triangle$ ABC : area of $\triangle$ DEC.*   **[3]**

(c) *Using graph paper, plot the points A(6, 4) and B(0, 4).*

   (i)   *Reflect A and B in the origin to get the images A' and B'.*

   (ii)   *Write the co-ordinates of A' and B'.*

   (iii)   *State the geometrical name for the figure ABA'B'.*

   (iv)   *Find its perimeter.*   **[4]**

Solution :

(a) *Given :* $x^2 - 5x - 10 = 0$

Here, $a = 1$, $b = -5$ and $c = -10$

$\therefore$

$$D = b^2 - 4ac$$

$$= (-5)^2 - 4 \times 1 \times -10$$

$$D = 25 + 40 = 65$$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$= \frac{5 \pm \sqrt{65}}{2 \times 1} = \frac{5 \pm 8 \cdot 06}{2}$$

$$= \frac{5 + 8 \cdot 06}{2}, \frac{5 - 8 \cdot 06}{2}$$

$$= \frac{13 \cdot 06}{2}, -\frac{3 \cdot 06}{2}$$

$$x = 6 \cdot 53, -1 \cdot 53 \qquad \textbf{Ans.}$$

(b) (i) From $\triangle ABC$ and $\triangle DEC$,

$$\angle ABC = \angle DEC = 90° \qquad \text{(Given)}$$

and

$$\angle ACB = \angle DCE = \text{Common}$$

$\therefore$

$$\triangle ABC \sim \triangle DEC \qquad \text{(By AA similarity)}$$

(ii) In $\triangle ABC$ and $\triangle DEC$,

$$\triangle ABC \sim \triangle DEC \qquad \text{(proved in (i) part)}$$

$\therefore$

$$\frac{AB}{DE} = \frac{AC}{CD}$$

Given : AB = 6 cm, DE = 4 cm, AC = 15 cm,

$\therefore$

$$\frac{6}{4} = \frac{15}{CD}$$

$\Rightarrow \qquad 6 \times CD = 15 \times 4$

$\Rightarrow \qquad CD = \dfrac{60}{6}$
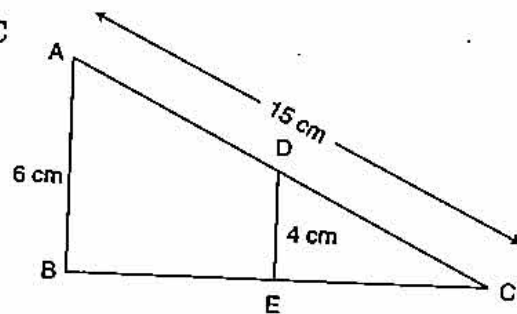
$\Rightarrow \qquad CD = 10 \text{ cm.} \qquad \textbf{Ans.}$

(iii)

$$\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle DEC} = \frac{AB^2}{DE^2} \qquad (\because \triangle ABC \sim \triangle DEC)$$

$$= \frac{(6)^2}{(4)^2}$$

$$= \frac{3 \cdot 6}{16} = \frac{9}{4}$$

$\therefore$ Area of $\triangle ABC$: Area of $\triangle DEC$ = 9 : 4. $\qquad \textbf{Ans.}$

(c) (i) Please See Graph:
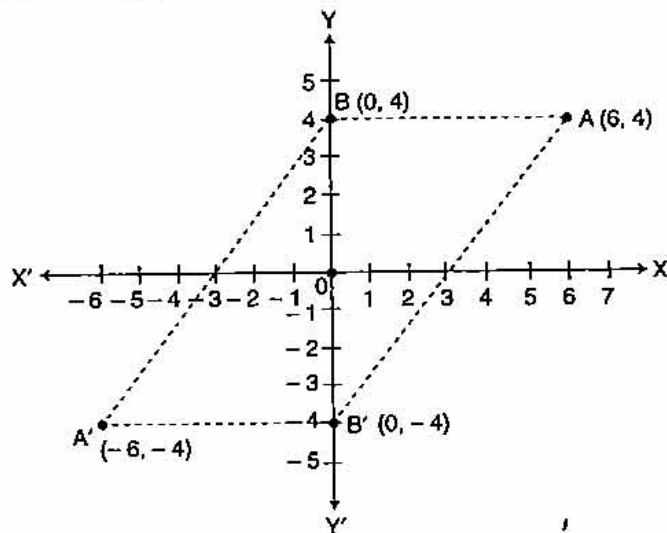
(ii) Reflection of A′ and B′ in the origin = A′ (–6, –4) and B′ (0, –4)

(iii) The geometrical name for the figure AB A′B′ is a parallelogram.

(iv) From the graph, AB = 6 cm, BB′ = 8 cm.

In Δ A BB′

$$(AB')^2 = AB^2 + (BB')^2$$
$$= (6)^2 + (8)^2 = 36 + 64$$
$$= 100$$
$$AB' = 10 = A'B \qquad \text{(AB A′ B′ is a parallelogram)}$$

Perimeter of AB A′B′ $= A'B' + AB' + AB + A'B$
$$= 6 + 10 + 6 + 10$$
$$= 32 \text{ units.} \qquad \textbf{Ans.}$$

SECTION B [40 Marks]

Answer any four Questions in this Section.

Question 5.

(a) *Solve the following inequation, write the solution set and represent it on the number line :*

$$-\frac{x}{3} \le \frac{x}{2} - 1\frac{1}{3} < \frac{1}{6}, x \in R \qquad \text{[3]}$$

(b) *Mr. Britto deposits a certain sum of money each month in a Recurring Deposit Account of a bank. If the rate of interest is of 8% per annum and Mr. Britto gets ₹ 8088 from the bank after 3 years, find the value of his monthly instalment.*

[3]

(c) *Salman buys 50 shares of face value ₹ 100 available at ₹ 132.*

(i) *What is his investment ?*

(ii) *If the dividend is 7.5%, what will be his annual income ?*

(iii) *If he wants to increase his annual income by ₹ 150, how many extra shares should he buy ?*
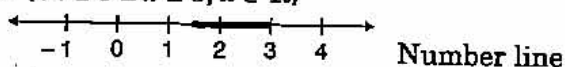
[4]

Solution :

(a) *Given :*
$$-\frac{x}{3} \le \frac{x}{2} - 1\frac{1}{3} < \frac{1}{6}$$

Taking L.C.M. of 3, 2 and 6 is 6.

$$-\frac{x}{3} \times 6 \le \frac{x}{2} \times 6 - \frac{4}{3} \times 6 < \frac{1}{6} \times 6$$

$$-2x \le 3x - 8 < 1$$

$\Rightarrow \qquad -2x \le 3x - 8 \qquad$ and $\qquad 3x - 8 < 1$

$\Rightarrow \qquad 8 \le 3x + 2x \qquad\qquad \Rightarrow \qquad 3x < 1 + 8$

$\Rightarrow \qquad 8 \le 5x \qquad\qquad\quad \Rightarrow \qquad 3x < 9$

$\Rightarrow \qquad \dfrac{8}{5} \le x \qquad\qquad\quad \Rightarrow \qquad x < 3$

∴ The solution set is $\{x : 1 \cdot 6 \le x \le 3, x \in R\}$

Number line

(b) Let the monthly instalment be ₹ x

Given : Maturity amount = ₹ 8,088, Time (n) = 3 years = 3 × 12 months = 36 months, Rate (R) = 8% p.a.

$$\text{Principle for one month} = P \times \frac{n(n+1)}{2}$$

$$= \frac{x \times 36 \times 37}{2}$$

$$= 18 \times 37\, x$$

$$\text{Interest} = \frac{18 \times 37x \times 8 \times 1}{100 \times 12} \qquad \left[\because I = \frac{PRT}{100} \right]$$

$$= \frac{444\, x}{100}$$

$$\text{Actual sum deposited} = 36\, x$$

$$\text{Maturity amount} = \text{Interest} + \text{Actual sum deposited}$$

$$8,088 = \frac{444\, x}{100} + 36\, x$$

$$8,088 = \frac{4,044\, x}{100}$$

$$\therefore \qquad x = \frac{8,088 \times 100}{4,044} = 200$$

Hence, the monthly instalment be ₹ 200. **Ans.**

(c)
$$\text{Number of shares} = 50$$
$$\text{Face value of each share} = ₹\ 100$$
$$\text{Market value of each share} = ₹\ 132$$
$$\text{Total face value} = ₹\ 100 \times 50$$
$$= ₹\ 5,000$$

(i)
$$\text{Total investment} = ₹\ 132 \times 50$$
$$= ₹\ 6,600 \qquad\qquad \textbf{Ans.}$$

(ii)

$$\text{Rate of dividend} = 7\cdot5\%$$

$$\text{Annual income} = ₹\frac{5,000 \times 7\cdot5}{100}$$

$$= ₹375 \qquad \textbf{Ans.}$$

(iii) Let extra share should he buy be x.

then total number of shares $= 50 + x$

$$\text{Total face value} = ₹100 \times (50 + x)$$

$$\therefore \quad \text{Annual income} = ₹\frac{100 \times (50 + x) \times 7\cdot5}{100}$$

$$= (50 + x) \times 7\cdot5$$

$$\therefore \quad (50 + x) \times 7\cdot5 = 375 + 150$$

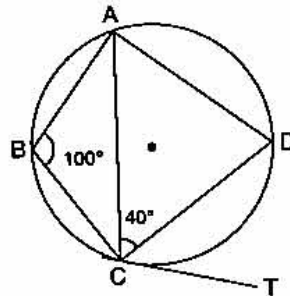$$50 + x = \frac{525}{7\cdot5} = 70$$

$$x = 70 - 50$$

$$x = 20$$

Hence, the extra shares should be buy $= 20$. **Ans.**

Question 6.

(a) Show that $\sqrt{\dfrac{1 - \cos A}{1 + \cos A}} = \dfrac{\sin A}{1 + \cos A}$ [3]

(b) In the given circle with centre O, $\angle ABC = 100°$, $\angle ACD = 40°$ and CT is a tangent to the circle at C. Find $\angle ADC$ and $\angle DCT$. [3]

(c) Given below are the entries in a Savings Bank A/c pass book :

Date	Particulars	Withdrawals	Deposit	Balance
Feb. 8.	B/F	—	—	₹ 8,500
Feb. 18	To self	₹ 4,000	—	—
April 12	By cash	—	₹ 2,230	—
June 15	To self	₹ 5,000	—	—
July 8	By cash	—	₹ 6,000	—

Calculate the interest for six months from February to July at 6% p.a. [4]

Solution :

(a) L.H.S. $= \sqrt{\dfrac{1 - \cos A}{1 + \cos A}}$

Multiplying by $\sqrt{1 + \cos A}$ in numerator and denominator

$$= \sqrt{\frac{1 - \cos A}{1 + \cos A}} \times \sqrt{\frac{1 + \cos A}{1 + \cos A}}$$

$$= \sqrt{\frac{(1 - \cos A)(1 + \cos A)}{(1 + \cos A)(1 + \cos A)}}$$

$$= \sqrt{\frac{1 - \cos^2 A}{(1 + \cos A)^2}}$$

$$= \sqrt{\frac{\sin^2 A}{(1 + \cos A)^2}}$$

$$= \frac{\sin A}{1 + \cos A} = \text{R.H.S.}$$

Proved

(b) *Given :* $\angle ABC = 100°$

We know that,

$$\angle ABC + \angle ADC = 180° \quad \text{(The sum of opposite angles in}$$

$$\therefore \quad 100° + \angle ADC = 180° \quad \text{a cyclic quadrilateral = 180°)}$$

$$\angle ADC = 180° - 100°$$

$$\angle ADC = 80°$$

Join OA and OC, we have a isosceles $\triangle$ OAC,

$$\therefore \quad OA = OC \quad \text{(Radii of a circle)}$$

$$\therefore \quad \angle AOC = 2 \times \angle ADC \quad \text{(by theorem)}$$

or

$$\angle AOC = 2 \times 80° = 160°$$

In $\triangle$ AOC,

$$\angle AOC + \angle OAC + \angle OCA = 180°$$

$$160° + \angle OCA + \angle OCA = 180° \quad [\because \angle OAC = \angle OCA]$$

$$2\angle OCA = 20°$$

$$\angle OCA = 10°$$

$$\angle OCA + \angle OCD = 40°$$

$$10° + \angle OCD = 40°$$

$$\therefore \quad \angle OCD = 30°$$
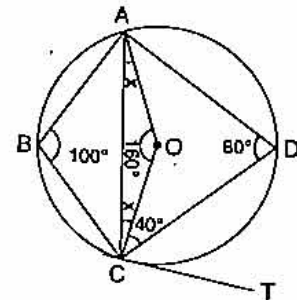
Hence, $\quad \angle OCD + \angle DCT = \angle OCT$

$$\therefore \quad \angle OCT = 90°$$

(The tangent at a point to circle is $\perp$ to the radius through the point to contant)

$$30° + \angle DCT = 90°$$

$$\therefore \quad \angle DCT = 60° \quad \text{Ans.}$$

(c)

Date	Particulars	Withdrawals	Deposit	Balance
Feb. 8	B/F	—	—	₹ 8,500
Feb. 18	To self	₹ 4,000	—	₹ 4,500
April 12	By cash	—	₹ 2,230	₹ 6,730
June 15	To self	₹ 5,000	—	₹ 1,730
July 8	By cash	—	₹ 6,000	₹ 7,730

Principal for the month of Feb. = ₹ 4,500

Principal for the month of March = ₹ 4,500

$$\text{Principal for the month of April} = ₹ \quad 4,500$$
$$\text{Principal for the month of May} = ₹ \quad 6,730$$
$$\text{Principal for the month of June} = ₹ \quad 1,730$$
$$\text{Principal for the month of July} = ₹ \quad 7,730$$
$$\text{Total principal from the month of Feb. to July} = ₹ \quad 29,690$$

$$\text{Time} = \frac{1}{12} \text{ years}$$

$$\text{Rate of interest} = 6\%$$

$$\text{Interest} = \frac{P \times R \times T}{100}$$

$$= \frac{29690 \times 6 \times 1}{100 \times 12}$$

$$= ₹ 148.45 \qquad \textbf{Ans.}$$

Question 7.

(a) In $\triangle ABC$, $A(3, 5)$, $B(7, 8)$ and $C(1, -10)$. Find the equation of the median through A. **[3]**

(b) A shopkeeper sells an article at the listed price of ₹ 1,500 and the rate of VAT is 12% at each stage of sale. If the shopkeeper pays a VAT of ₹ 36 to the Government, what was the price, inclusive of Tax, at which the shopkeeper purchased the article from the wholesaler ? **[3]**

(c) In the figure given, from the top of a building $AB = 60$ m high, the angles of depression of the top and bottom of a vertical lamp post CD are observed to be 30° and 60° respectively. Find :

    (i)   The horizontal distance between AB and CD.
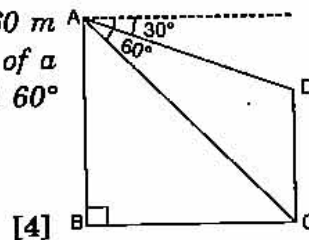
    (ii)   The height of the lamp post. **[4]**

Solution :

(a) Here D is mid point of BC.

$$\therefore \qquad \text{The co-ordinate of D} = \left(\frac{7+1}{2}, \frac{8-10}{2}\right)$$
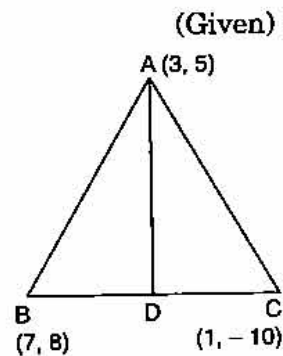
$$= (4, -1)$$

Now equation of median AD,

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Here, $x_1 = 3$, $y_1 = 5$, $x_2 = 4$, $y_2 = -1$

$$y - 5 = \frac{-1-5}{4-3}(x-3)$$

$$y - 5 = \frac{-6}{1}(x-3)$$

$$y - 5 = -6x + 18$$

$$y = -6x + 18 + 5$$
$$y = -6x + 23$$
$$6x + y - 23 = 0 \qquad \text{**Ans.**}$$

(b)

Listed price of an article $= ₹ 1,500$

Rate of VAT $= 12\%$

VAT on the article $= \dfrac{12}{100} \times 1500$

$$= ₹ 180$$

Let C.P. of this article be x, then

$$\text{VAT} = \frac{12}{100} \times x$$

$$= ₹ \frac{12x}{100}$$

If the shopkeeper pays a VAT $= ₹ 36$

Then

$$180 - \frac{12x}{100} = 36$$

$$\frac{18000 - 12x}{100} = 36$$

$$18000 - 12x = 3600$$

$\therefore \qquad 12x = 18000 - 3600 = 14,400$

$$x = ₹ 1,200$$

$\therefore$ The price at which the shopkeeper purchased the article inclusive of sales tax

$$= 1,200 + \frac{12}{100} \times 1,200$$

$$= 1,200 + 144$$

$$= ₹ 1,344 \qquad \text{**Ans.**}$$

(c) *Given* : AB = 60 m

$\therefore$

$\angle\, \text{PAC} = 60°$

$\angle\, \text{PAC} = \angle\, \text{BCA}$
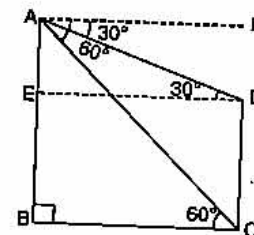
(i) Now in $\triangle$ ABC,

$$\tan 60° = \frac{AB}{BC}$$

$$\sqrt{3} = \frac{60}{BC}$$

$\Rightarrow \qquad \sqrt{3}\; BC = 60$

$\Rightarrow \qquad BC = \dfrac{60}{\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}}$

$$BC = \frac{60\sqrt{3}}{3} = 20\sqrt{3}$$

Hence, the horizontal distance between AB and CD = $20\sqrt{3}$ m. **Ans.**

(ii) Let AE = x and proved above BC = $20\sqrt{3}$ m

$\therefore \qquad BC = ED = 20\sqrt{3}$

Now in Δ AED,

$$\tan 30° = \frac{AE}{ED}$$

$$\frac{1}{\sqrt{3}} = \frac{AE}{20\sqrt{3}}$$

$$\Rightarrow \qquad \sqrt{3}\,AE = 20\sqrt{3}$$

$$\Rightarrow \qquad AE = 20 \text{ m}$$

now $\qquad EB = AB - AE$

∴ $\qquad EB = 60 - 20 \qquad \Rightarrow \qquad 40 \text{ m}$

∵ $\qquad EB = CD$

∴ $\qquad CD = 40 \text{ m}$

Hence, the height of the lamp post = 40 m. **Ans.**

Question 8.

(a) *Find x and y if* $\begin{bmatrix} x & 3x \\ y & 4y \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$ [3]

(b) *A solid sphere of radius 15 cm is melted and recast into solid right circular cones of radius 2.5 cm and height 8 cm. Calculate the number of cones recast.* [3]

(c) *Without solving the following quadratic equation, find the value of 'p' for which the given equation has real and equal roots :*

$$x^2 + (p - 3)\,x + p = 0 \qquad \text{[4]}$$

Solution :

(a) *Given :* $\begin{bmatrix} x & 3x \\ y & 4y \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$

$$\begin{bmatrix} 2x + 3x \\ 2y + 4y \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} 5x \\ 6y \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$$

∴ $\qquad 5x = 5 \Rightarrow x = 1$

and $\qquad 6y = 12 \Rightarrow y = 2$

Hence, $x = 1$ and $y = 2$ **Ans.**

(b) $\qquad$ Radius of a solid sphere, $r = 15$ cm

$$\text{Volume of a solid sphere} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \pi\,(15)^3 \text{ cm}^3.$$

Now, $\qquad$ radius of right circular cone = 2·5 cm

and $\qquad$ height, $h = 8$ cm.

$$\text{Volume of right circular cone} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi\,(2\cdot5)^2 \times 8$$

∴ The number of cones $= \dfrac{\text{Volume of a sphere}}{\text{Volume of a cone}}$

$$= \dfrac{\frac{4}{3}\pi \times (15)^3}{\frac{1}{3}\pi(2\cdot5)^2 \times 8}$$

$$= \dfrac{15 \times 15 \times 15}{2\cdot5 \times 2\cdot5 \times 2}$$

$$= 270 \qquad \textbf{Ans.}$$

(c) Given equation $\qquad x^2 + (p-3)x + p = 0$

∵ Roots are real and equal, then

$$b^2 - 4ac = 0$$

Here we compare the coefficients of a, b and c with the equation $ax^2 + bx + c = 0$.

$$a = 1, b = p - 3 \text{ and } c = p$$

Now putting the values of a, b and c in equation

$$(p-3)^2 - 4 \times 1 \times p = 0$$

$$p^2 + 9 - 6p - 4p = 0$$

$$p^2 + 9 - 10p = 0$$

$$p^2 - 10p + 9 = 0$$

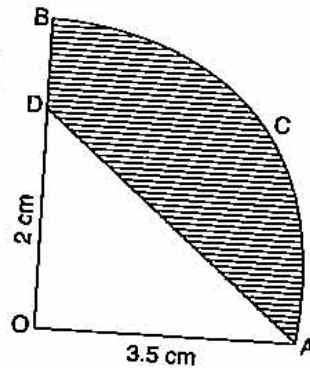$$p^2 - 9p - p + 9 = 0$$

$$p(p-9) - 1(p-9) = 0$$

$$\Rightarrow \qquad (p-9)(p-1) = 0$$

Hence,

$$p = 9 \text{ or } 1 \qquad \textbf{Ans.}$$

Question 9.

(a) *In the figure alongside, OAB is a quadrant of a circle. The radius OA = 3.5 cm and OD = 2 cm. Calculate the area of the shaded portion.* $\left(\text{Take } \pi = \dfrac{22}{7}\right)$ **[3]**

(b) *A box contains some black balls and 30 white balls. If the probability of drawing a black ball is two-fifths of a white ball, find the number of black balls in the box.* **[3]**

(c) *Find the mean of the following distribution by step deviation method :* **[4]**

Class Interval	20–30	30–40	40–50	50–60	60–70	70–80
Frequency	10	6	8	12	5	9

Solution :

(a) Radius of quadrant OACB, $r = 3.5$ cm

$\therefore$ Area of quadrant OACB $= \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \frac{22}{7} \times 3.5 \times 3.5$

$= 9.625$ cm^2.

Here, $\angle AOD = 90°$

Then area of Δ AOD $= \frac{1}{2} \times$ base $\times$ height

Base $= 3.5$ cm and height $= 2$ cm

$\therefore \quad = \frac{1}{2} \times 3.5 \times 2 = 3.5$ cm^2.

Area of shaded portion $=$ Area of quadrant $-$ Area of triangle

$= 9.625 - 3.5$

$= 6.125$ cm^2. **Ans.**

(b) Let the number of black balls be x, then

Total number of balls $= 30 + x$

Thus, the probability of black balls $= \frac{x}{30+x}$

and the probability of white balls $= \frac{30}{30+x}$

Given : Probability of black ball $= \frac{2}{5} \times$ probability of white ball

$\frac{x}{30+x} = \frac{2}{5} \times \frac{30}{x+30}$

$5x = 60$

$x = 12$ **Ans.**

Hence, the number of black balls $= 12$.

(c)

C.I.	Frequency (f_i)	Mid-value (x)	$d_i = \frac{x-a}{h}$	$f_i d_i$
20–30	10	25	−2	−20
30–40	6	35	−1	−6
40–50	8	45	0	0
50–60	12	55	1	12
60–70	5	65	2	10
70–80	9	75	3	27
	$\Sigma f_i = 50$			$\Sigma f_i d_i = 23$

Here, $a = 45$ and $h = 10$

$\therefore$ Mean $= a + \frac{\Sigma f_i d_i}{\Sigma f_i} \times h$

$= 45 + \frac{23}{50} \times 10$

$= 45 + 4.6 = 49.6$. **Ans.**

Question 10.

(a) *Using a ruler and compasses only :*

(i) *Construct a triangle ABC with the following data :*
AB = 3.5 cm, BC = 6 cm and ∠ABC = 120°

(ii) *In the same diagram, draw a circle with BC as diameter. Find a point P on the circumference of the circle which is equidistant from AB and BC.*

(iii) *Measure ∠BCP.*   **[3]**

(b) *The mark obtained by 120 students in a test are given below :*

Marks	No. of Students
0–10	5
10–20	9
20–30	16
30–40	22
40–50	26
50–60	18
60–70	11
70–80	6
80–90	4
90–100	3

Draw an ogive for the given distribution on a graph sheet.

Using suitable scale for ogive to estimate the following :

(i) *The median.*

(ii) *The number of students who obtained more than 75% marks in the test.*

(iii) *The number of students who did not pass the test if minimum marks required to pass is 40.*   **[6]**

Solution :

(a) **Steps of Construction :**

(i) Draw a line BC = 6 cm.

(ii) With the help of the point B, draw ∠ ABC = 120°
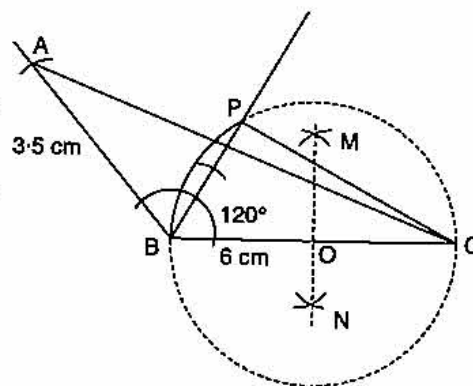
(iii) Taking radius 3·5 cm cut BA = 3·5 cm.

(iv) Join A to C.

(v) Draw ⊥ bisector MN of BC.

(vi) Draw a circle O as centre and OC as radius.

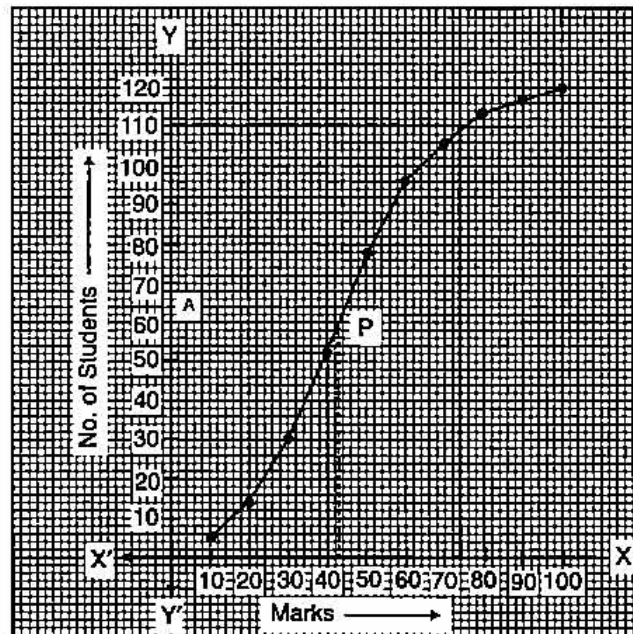(vii) Draw angle bisector of ∠ ABC which intersects circle at P.

(viii) Join BP and CP.

(ix) Now, ∠ BCP = 30°.

(b)

Marks	No. of Students (f)	Cumulative Frequency
0–10	5	5
10–20	9	14
20–30	16	30
30–40	22	52
40–50	26	78
50–60	18	96
60–70	11	107
70–80	6	113
80–90	4	117
90–100	3	120
	$n = 120$	

On the graph paper, we plot the following points :

(10, 5), (20, 14), (30, 30), (40, 52), (50, 78), (60, 96), (70, 107), (80, 113), (90, 117), (100, 120).

(i) $$\text{Median} = \left(\frac{n}{2}\right)^{\text{th}} \text{ term} \qquad [\because n = 120, \text{ even}]$$

$$= \frac{120}{2} = 60^{\text{th}} \text{ term}$$

From the graph 60th term = 42 **Ans.**

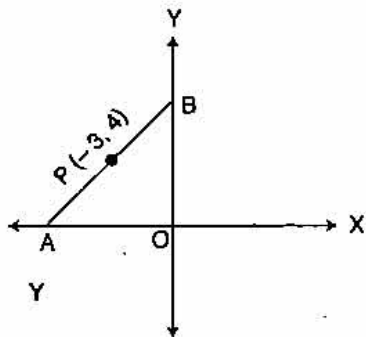(ii) The number of students who obtained more than 75% marks in test
$$= 120 - 110$$
$$= 10. \qquad \textbf{Ans.}$$

(iii) The number of students who did not pass the test if the minimum pass marks 40 = 52. **Ans.**

Question 11.

(a) *In the figure given below, the line segment AB meets X-axis at A and Y-axis at B. The point P(–3, 4) on AB divides it in the ratio 2 : 3. Find the coordinates of A and B.*

(b) *Using the properties of proportion, solve for x, given*

$$\frac{x^4 + 1}{2x^2} = \frac{17}{8}$$

[3]

(c) *A shopkeeper purchases a certain number of books for ₹ 960. If the cost per book was ₹ 8 less, the number of books that could be purchased for ₹ 960 would be 4 more. Write an equation, taking the original cost of each book to be ₹ x, and solve it to find the original cost of the books.* [4]

Solution :

(a) Let the co-ordinates of A and B be $(x, 0)$ and $(0, y)$

∵ The co-ordinates of a point P $(-3, 4)$ on AB divides it in the ratio 2 : 3.

i.e., AP : PB = 2 : 3

By using section formula, we get

$$-3 = \frac{2 \times 0 + 3 \times x}{2 + 3} \qquad \left[\because x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2} \right]$$

$$-3 = \frac{3x}{5} \quad \Rightarrow \quad 3x = -15$$

$$\Rightarrow \qquad x = -5$$

and $$4 = \frac{2 \times y + 3 \times 0}{2 + 3} \qquad \left[\because y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} \right]$$

$$4 = \frac{2y}{5} \quad \Rightarrow \quad 2y = 20$$

$$\Rightarrow \qquad y = 10$$

Hence, the co-ordinates of A and B are (–5, 0) and (0, 10). **Ans.**

(b) *Given :* $$\frac{x^4 + 1}{2x^2} = \frac{17}{8}$$

By using componendo and dividendo, we get

$$\frac{x^4 + 1 + 2x^2}{x^4 + 1 - 2x^2} = \frac{17 + 8}{17 - 8}$$

$$\left(\frac{x^2+1}{x^2-1}\right)^2 = \frac{25}{9}$$

$$\left(\frac{x^2+1}{x^2-1}\right)^2 = \left(\frac{5}{3}\right)^2$$

Taking square root on both sides, we get

$$\frac{x^2+1}{x^2-1} = \frac{5}{3}$$

$\Rightarrow \qquad 5x^2 - 5 = 3x^2 + 3$

$\Rightarrow \qquad 5x^2 - 3x^2 = 3 + 5$

$\Rightarrow \qquad 2x^2 = 8 \quad \Rightarrow \quad x^2 = 4$

$\Rightarrow \qquad x = \pm 2$ **Ans.**

(c) Given the original cost of each book be ₹ x.

Total cost = ₹ 960 (Given)

∴ Number of books for 960 = $\dfrac{960}{x}$

If the cost per book was ₹ 8 less, (*i.e.*, $x - 8$) then

Number of books = $\dfrac{960}{x-8}$

According to question,

$$\frac{960}{x-8} = \frac{960}{x} + 4$$

$$\frac{960}{x-8} - \frac{960}{x} = 4$$

$$960\left[\frac{x-x+8}{x(x-8)}\right] = 4$$

$$\frac{8}{x^2-8x} = \frac{1}{240}$$

$\Rightarrow \qquad x^2 - 8x = 1{,}920$

$\Rightarrow \qquad x^2 - 8x - 1{,}920 = 0$ **Ans.**

$\Rightarrow \qquad x^2 - 48x + 40x - 1{,}920 = 0$

$\Rightarrow \qquad x(x-48) + 40(x-48) = 0$

$\Rightarrow \qquad (x-48)(x+40) = 0$

$x - 48 = 0 \qquad$ or $\;x + 40 = 0$

$x = 48 \qquad$ or $\qquad x = -40$

∵ -40 is not possible.

Hence, the original cost of each book = ₹ 48. **Ans.**

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