ICSE Paper 2011

MATHEMATICS

SECTION A [40 Marks]

*(Answer **all** questions from this Section.)*

Question 1.

(a) *Find the value of 'k' if $(x-2)$ is a factor of :*

$$x^3 + 2x^2 - kx + 10$$

Hence determine whether $(x+5)$ is also a factor. **[3]**

(b) *If* $A = \begin{bmatrix} 3 & 5 \\ 4 & -2 \end{bmatrix}$ *and* $B = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$, *is the product AB possible ? Give a reason. If yes, find AB.* **[3]**

(c) *Mr. Kumar borrowed ₹ 25,000 for two years. The rate of interest for the two successive years are 8% and 10% respectively. If he repays ₹ 6,200 at the end of the first year, find the outstanding amount at the end of the second year.* **[4]**

Solution :

(a) Let $f(x) = x^3 + 2x^2 - kx + 10$

$\because (x-2)$ is a factor,

$\therefore \quad f(2) = 0$

$f(2) = 8 + 8 - 2k + 10 = 0$

$\Rightarrow \quad k = 13$

To check for $(x+5)$ is a factor,

$$f(-5) = (-5)^3 + 2(-5)^2 - 13(-5) + 10$$
$$= -125 + 50 + 65 + 10 = 0$$

$\therefore (x+5)$ is a factor.

(b) $A_{2\times 2} \cdot B_{2\times 1}$

From the order of both the matrix A and B, it is clear that AB is possible because the number of columns of A are equal to the number of rows of B.

$$AB = \begin{bmatrix} 3 & 5 \\ 4 & -2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 6 + 20 \\ 8 - 8 \end{bmatrix}$$

$$= \begin{bmatrix} 26 \\ 0 \end{bmatrix}$$ **Ans.**

(c) *Given* : Principal = ₹ 15,000

We know that $A = P\left(1 + \frac{r}{100}\right)^n$

$$\text{Amount after } 1^{st} \text{ year} = 15{,}000\left(1 + \frac{8}{100}\right)$$
$$= ₹\ 16{,}200$$

Principal after repayment $= 16{,}200 - 6{,}200 = ₹\ 10{,}000$

Amount outstanding at the end of second year

$$= 10{,}000\left(1 + \frac{10}{100}\right)$$
$$= ₹\ 11{,}000$$

Ans.

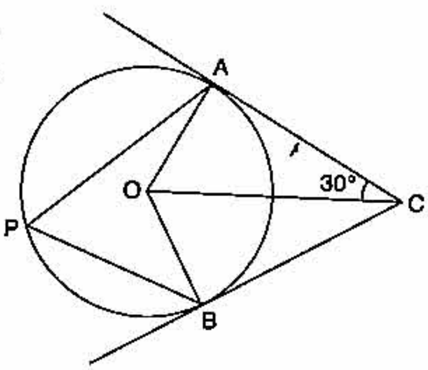
Question 2.

(a) *From a pack of 52 playing cards all cards whose numbers are multiples of 3 are removed. A card is now drawn at random.*
What is the probability that the card drawn is :
(i) *a face card (King, Jack or Queen)*
(ii) *an even numbered red card ?* **[3]**

(b) *Solve the following equation :*

$x - \frac{18}{x} = 6$. *Give your answer correct to two significant figures.* **[3]**

(c) *In the given figure O is the centre of the circle. Tangents at A and B meet at C. If* $\angle AOC = 30°$, *find*
(i) $\angle BCO$
(ii) $\angle AOB$
(iii) $\angle APB$ **[4]**



Solution :

(a) Number of cards which are multiples of 3 = 12

Cards left in the pack = 40

(i) Number of face cards = 12

$$P(\text{face card}) = \frac{12}{40} = \frac{3}{10}$$ **Ans.**

(ii) Even numbered red cards = 10

$$P(\text{even number red card}) = \frac{10}{40} = \frac{1}{4}$$ **Ans.**

(b) Let $x^2 - 6x - 18 = 0$

Compare with equation $ax^2 + bx + c = 0$, we get
$a = 1, b = -6, c = -18$

Now,
$$x = \frac{b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{6 \pm \sqrt{36 + 72}}{2}$$

$$= \frac{6 \pm 6\sqrt{3}}{2} \Rightarrow x = 3 \pm 3\sqrt{3}$$

$\therefore$ $$x = 3 \pm 5{\cdot}196$$

Taking $+ve$ and $-ve$ sign respectively, we get

$$x = 8{\cdot}196 \text{ or } x = -2{\cdot}196$$ **Ans.**

(c) $\Delta ACO \cong \Delta BCO$ (R.H.S.)

$\therefore$ $\angle BCO = \angle ACO$ (C.P.C.T.)

(i) $\angle BCO = 30°$

In ΔACO, $\angle OAC = 90°$ (Radius is perpendicular to tangent)

$\therefore$ $\angle AOC = 60°$

Also $\angle BOC = 60°$ (C.P.C.T.)

(ii) $\angle AOB = 120°$

(iii) $\angle APB = 60°$ (Angle at circumference is half the angle at the centre)

Question 3.

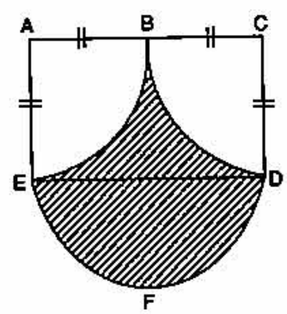
(a) *Ahmed has a recurring deposit account in a bank. He deposits ₹ 2,500 per month for 2 years. If he gets ₹ 66,250 at the time of maturity, find*

(i) *The interest paid by the bank.*

(ii) *The rate of interest.* **[3]**

(b) *Calculate the area of the shaded region, if the diameter of the semi circle is equal to 14 cm.*

Take $\pi = \frac{22}{7}$ **[3]**



(c) *ABC is a triangle and G(4, 3) is the centroid of the triangle. If A = (1, 3), B = (4, b) and C = (a, 1), find 'a' and 'b'. Find the length of side BC.* **[4]**

Solution :

(a) (i) Interest $= 66{,}250 - 2{,}500 \times 24$

$= 66{,}250 - 60{,}000$

$=$ ₹ 6,250 **Ans.**

(ii) Principal $= \frac{n(n+1)}{2} \times$ sum of deposited per month

$$= \frac{24(24+1)}{2} \times 2{,}500$$

$$= \frac{24 \times 25}{2} \times 2{,}500$$

$$I = \text{Principal} \times \frac{R}{100} \times \frac{1}{12}$$

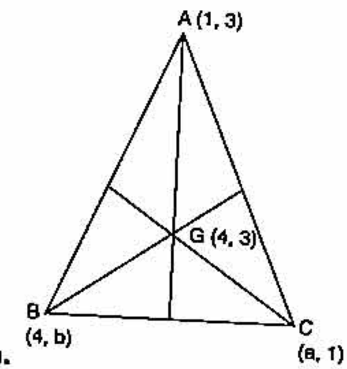
$$R = \frac{6{,}250 \times 2 \times 100 \times 12}{2{,}500 \times 24 \times 25}$$
$$= 10\% \text{ p.a.}$$
Ans.

(b) Area of shaded portion $=$ Complete area $-$ area of the two quadrants

$=$ (Area of ACDE + Area of semi circle EFD) $-$ (Area of Quadrant ABE + Area of Quadrant BCD)

$$= \left\{14 \times 7 + \frac{\pi}{2}(7)^2\right\} - \left\{\frac{\pi}{4}(7)^2 + \frac{\pi}{4}(7)^2\right\}$$
$$= \left\{14 \times 7 + \frac{\pi}{2}(7)^2\right\} - \left\{\frac{\pi}{2}(7)^2\right\}$$
$$= 98 \text{ cm}^2.$$
Ans.

(c) Coordinate of centroid $G\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$



$$\Rightarrow \quad \frac{1 + 4 + a}{3} = 4 \quad \Rightarrow \quad a = 7$$
$$\frac{3 + b + 1}{3} = 3 \quad \Rightarrow \quad b = 5$$

Now, $\quad BC = \sqrt{(4-7)^2 + (5-1)^2}$

$= \sqrt{9 + 16} = 5$ units. **Ans.**

Question 4.

(a) *Solve the following inequation and represent the solution set on the number line* $2x - 5 \le 5x + 4 < 11$*, where* $x \in I$.

(b) *Evaluate without using trigonometric tables :* **[3]**

$$2\left(\frac{\tan 35^\circ}{\cot 55^\circ}\right)^2 + \left(\frac{\cot 55^\circ}{\tan 35^\circ}\right)^2 - 3\left(\frac{\sec 40^\circ}{\operatorname{cosec} 50^\circ}\right)$$
[3]

(c) *A Mathematics aptitude test of 50 students was recorded as follows :*

Marks	*50-60*	*60-70*	*70-80*	*80-90*	*90-100*
No. of students	*4*	*8*	*14*	*19*	*5*

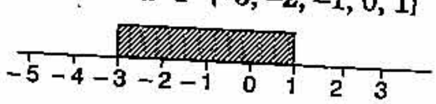
Draw a histogram for the above data using a graph paper and locate the mode.

Solution : **[4]**

(a) *Given :*

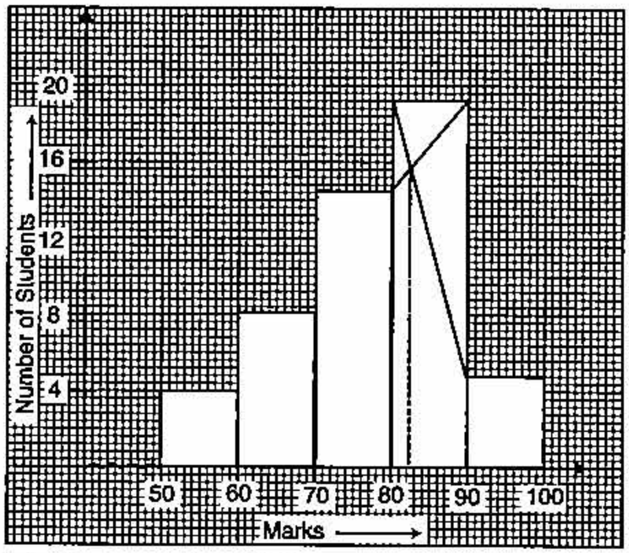
$$2x - 5 \le 5x + 4 \quad \text{and} \quad 5x + 4 < 11$$
$$-3x \le 9 \qquad\qquad 5x < 7$$
$$x \ge -3 \qquad\qquad x < 1{\cdot}4$$
$$-3 \le x$$

Solution set, $\quad x \in \{-3, -2, -1, 0, 1\}$



(b) *Given :*
$$2\left(\frac{\tan 35^\circ}{\cot 55^\circ}\right)^2 + \left(\frac{\cot 55^\circ}{\tan 35^\circ}\right) - 3\left(\frac{\sec 40^\circ}{\text{cosec } 50^\circ}\right)$$
$$= 2\left(\frac{\tan (90^\circ - 55^\circ)}{\cot 55^\circ}\right)^2 + \left(\frac{\cot (90^\circ - 35^\circ)}{\tan 35^\circ}\right) - 3\left(\frac{\sec (90^\circ - 50^\circ)}{\text{cosec } 50^\circ}\right)$$
$$= 2\left(\frac{\cot 55^\circ}{\cot 55^\circ}\right)^2 + \left(\frac{\tan 35^\circ}{\tan 35^\circ}\right) - 3\left(\frac{\text{cosec } 50^\circ}{\text{cosec } 50^\circ}\right)$$
$$= 2 + 1 - 3 = 0$$
Ans.

(c)



Mode from graph = 82·5.

SECTION B [40 Marks]

*Answer any **four** Questions in this Section.*

Question 5.

(a) *A manufacturer sells a washing machine to a wholesaler for ₹ 15,000. The wholesaler sells it to a trader at a profit of ₹ 1,200 and the trader in turn sells it to a consumer at a profit of ₹ 1,800. If the rate of VAT is 8% find :*

(i) *The amount of VAT received by the State Government on the sale of this machine from the manufacturer and the wholesaler.*

(ii) *The amount that the consumer pays for the machine.* **[3]**

(b) *A solid cone of radius 5 cm and height 8 cm is melted and made into small spheres of radius 0.5 cm. Find the number of spheres formed.* **[3]**

(c) *ABCD is a parallelogram where A(x, y), B (5, 8), C (4, 7) and D (2, –4). Find*

(i) *Coordinates of A*

(ii) *Equation of diagonal BD.* **[4]**

Solution :

(a) (i) VAT received by Govt. from manufacturer $= 15{,}000 \times \frac{8}{100} = ₹\ 1{,}200$

VAT from wholesaler $= 1200 \times \frac{8}{100} = ₹\ 96$

Total VAT from manufacturer and wholesaler

$= 1200 + 96 = ₹\ 1296$ **Ans.**

(ii) Amount that customer pays $= (15000 + 1200 + 1800) + \text{VAT}$

$$= 18{,}000 + \frac{18{,}000 \times 8}{100}$$

$$= 18{,}000 + 1440 = ₹\,19{,}440$$ **Ans.**

(b)

$$\text{Number of spheres} = \frac{\text{Volume of cone}}{\text{Volume of each sphere}}$$

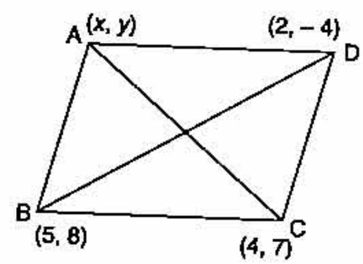
$$= \frac{\frac{1}{3}\pi (5)^2 (8)}{\frac{4}{3}\pi (0 \cdot 5)^3} = \frac{50 \times 10^3}{5 \times 5 \times 5}$$

$$= 400$$ **Ans.**

(c) In a parallelogram, mid point of diagonal BD co-incides with the mid point of diagonal AC.

$$\text{Mid point of BD} = \left(\frac{5+2}{2}, \frac{8-4}{2}\right) = \left(\frac{7}{2}, 2\right)$$

$$\text{Mid point of AC} = \left(\frac{x+4}{2}, \frac{y+7}{2}\right)$$



Equating, $\frac{x+4}{2} = \frac{7}{2} \Rightarrow x = 3$

$\frac{y+7}{2} = 2 \Rightarrow y = -3$

(i) Co-ordinates of A (3, –3). **Ans.**

(ii)

$$m \text{ of BD} = \frac{8-(-4)}{5-2} = \frac{12}{3} = 4$$

Equation of BD,

$$y - y_1 = m(x - x_1)$$

$$y + 4 = 4(x - 2)$$

$$\Rightarrow \quad y = 4x - 4$$ **Ans.**

Question 6.

(a) *Use a graph paper to answer the following questions. (Take 1 cm = 1 unit on both axes).*

(i) *Plot A(4, 4), B (4, –6) and C (8, 0), the vertices of a triangle ABC.*

(ii) *Reflect ABC on the y-axis and name it as A′B′C′.*

(iii) *Write the coordinates of the image A′, B′ and C′.*

(iv) *Give a geometrical name for the figure AA′C′B′BC.*

(v) *Identify the line of symmetry of AA′C′B′BC.* **[5]**

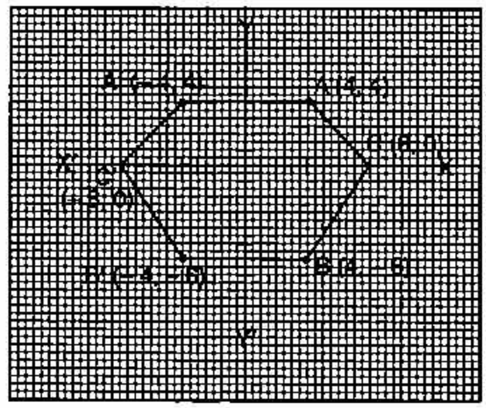
(b) *Mr. Choudhury opened a Saving's Bank Account at State Bank of India on 1st April 2007. The entries of one year as shown in his pass book are given below :*

Date	*Particulars*	*Withdrawals (in ₹)*	*Deposits (in ₹)*	*Balance (in ₹)*
1st April 2007	*By Cash*	—	*8550.00*	*8550.00*
12th April 2007	*To Self*	*1200.00*	—	*7350.00*
24th April 2007	*By Cash*	—	*4550.00*	*11900.00*

8th July 2007	*By Cheque*	—	*1500.00*	*13400.00*
10th Sept. 2007	*By Cheque*	—	*3500.00*	*16900.00*
17th Sept. 2007	*To Cheque*	*2500.00*	—	*14400.00*
11th Oct. 2007	*By Cash*	—	*800.00*	*15200.00*
6th Jan. 2008	*To Self*	*2000.00*	—	*13200.00*
9th March 2008	*By Cheque*	—	*950.00*	*14150.00*

If the bank pays interest at the rate of 5% per annum, find the interest paid on 1st April, 2008. Give your answer correct to the nearest rupee. **[5]**

Solution :



(a) (i) In the given diagram.

(ii) In the given diagram.

(iii) A′ (–4, 4)

B′ (–4, –6)

C′ (–8, 0)

(iv) Hexagon.

(v) Y–Axis is the line of symmetry.

(b) **Minimum Balances**

April	7,350.00
May	11,900.00
June	11,900.00
July	13,400.00
August	13,400.00
September	14,400.00
October	14,400.00
November	15,200.00
December	15,200.00
January	13,200.00
February	13,200.00
March	14,150.00
	1,57,700.00

$\Rightarrow$ P = ₹ 1,57,700

$$I = \frac{1,57,700 \times 5}{100} \times \frac{1}{12}$$

$$= \frac{7885}{12} = ₹\ 657.08$$ **Ans.**

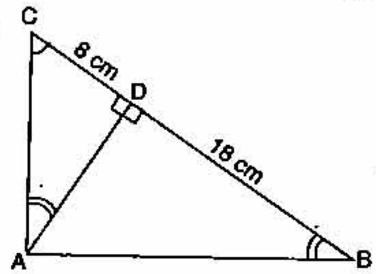
Question 7.

(a) *Using componendo and dividendo, find the value of x*

$$\frac{\sqrt{3x+4}+\sqrt{3x-5}}{\sqrt{3x+4}-\sqrt{3x-5}} = 9$$ **[3]**

(b) *If* $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$ *and I is the identity matrix of the same order and* A^t *is the transpose of matrix A, find* $A^t.B + BI$. **[3]**

(c) *In the adjoining figure ABC is a right angled triangle with* $\angle BAC = 90°$.



(i) *Prove* $\Delta ADB \sim \Delta CDA$.

(ii) *If BD = 18 cm, CD = 8 cm, find AD.*

(iii) *Find the ratio of the area of* Δ *ADB is to area of* Δ *CDA.* **[4]**

Solution :

(a) *Given :*
$$\frac{\sqrt{3x+4}+\sqrt{3x-5}}{\sqrt{3x+4}-\sqrt{3x-5}} = \frac{9}{1}$$

Applying componendo and Dividendo,
$$\frac{\sqrt{3x+4}+\sqrt{3x-5}+\sqrt{3x+4}-\sqrt{3x-5}}{\sqrt{3x+4}+\sqrt{3x-5}-\sqrt{3x+4}+\sqrt{3x-5}} = \frac{9+1}{9-1}$$
$$\frac{2\sqrt{3x+4}}{2\sqrt{3x-5}} = \frac{10}{8}$$
$$\frac{\sqrt{3x+4}}{\sqrt{3x-5}} = \frac{5}{4}$$

Squaring both sides,
$$\frac{3x+4}{3x-5} = \frac{25}{16}$$

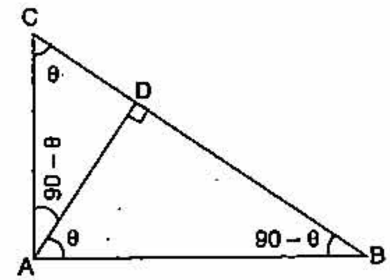
Applying Componendo and Dividendo,
$$\frac{3x+4+3x-5}{3x+4-3x+5} = \frac{25+16}{25-16}$$
$$\frac{6x-1}{9} = \frac{41}{9}$$
$$6x = 42$$
$$x = 7$$
Ans.

(b) Transpose of matrix A,
$$A^t = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$$
$$A^t \cdot B = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} \cdot \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$$
$$= \begin{bmatrix} 8-1 & -4+4 \\ 20-3 & -10+9 \end{bmatrix} = \begin{bmatrix} 7 & -1 \\ 17 & -1 \end{bmatrix}$$
$$B.I = \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$$

$$A^t \cdot B + B.I = \begin{bmatrix} 7 & -1 \\ 17 & -1 \end{bmatrix} + \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & -3 \\ 6 & 2 \end{bmatrix}$$ **Ans.**

(c) Let $\angle DAB = \theta$



$\therefore$ $\angle DAC = 90 - \theta$

$\angle DBA = 90 - \theta$

$\angle DCA = \theta$

$\because$ All three angles of Δ ADB are equal to all angles of Δ CDA.

(i) $\therefore$ $\Delta ADB \sim \Delta CDA$ **Proved**

(ii) $\therefore$ $\dfrac{CD}{AD} = \dfrac{AD}{BD}$

$\Rightarrow$ $AD^2 = CD \times BD$

$= 8 \times 18 \Rightarrow AD = 12$ **Ans.**

(iii) $$\frac{\Delta ADB}{\Delta CDA} = \frac{\frac{1}{2} AD \times BD}{\frac{1}{2} AD \times CD}$$

$$= \frac{BD}{CD} = \frac{18}{8}$$

$$= \frac{9}{4}$$ **Ans.**

Question 8.

(a) (i) *Using step-deviation method, calculate the mean marks of the following distribution.*

(ii) *State the modal class :* **[5]**

Class Interval	Frequency
50–55	5
55–60	20
60–65	10
65–70	10
70–75	9
75–80	6
80–85	12
85–90	8

(b) *Marks obtained by 200 students in an examination are given below :*

Marks	No. of Students
0–10	5
10–20	11
20–30	10
30–40	20
40–50	28
50–60	37
60–70	40
70–80	29
80–90	14
90–100	6

Draw an ogive for the given distribution taking 2 cm = 10 marks on one axis and 2 cm = 20 students on the other axis. Using the graph, determine

(i) *The median marks.*

(ii) *The number of students who failed if minimum marks required to pass is 40.*

(iii) *If scoring 85 and more marks is considered as grade one, find the number of students who secured grade on in the examination.* **[5]**

Solution :

(a) (i)

C.I.	f	x	$d = x - 67{\cdot}5$	u	$f.u$
50–55	5	52·5	–15	–3	–15
55–60	20	57·5	–10	–2	–40
60–65	10	62·5	–5	–1	–10
65–70	10	67·5	0	0	0
70–75	9	72·5	5	1	9
75–80	6	77·5	10	2	12
80–85	12	82·5	15	3	36
85–90	8	87·5	20	4	32
	$\Sigma f = 80$				$\Sigma fu = 24$

A.M. = 67·5

$$\bar{x} = \text{A.M.} + \frac{\Sigma fu}{\Sigma f} \times i$$

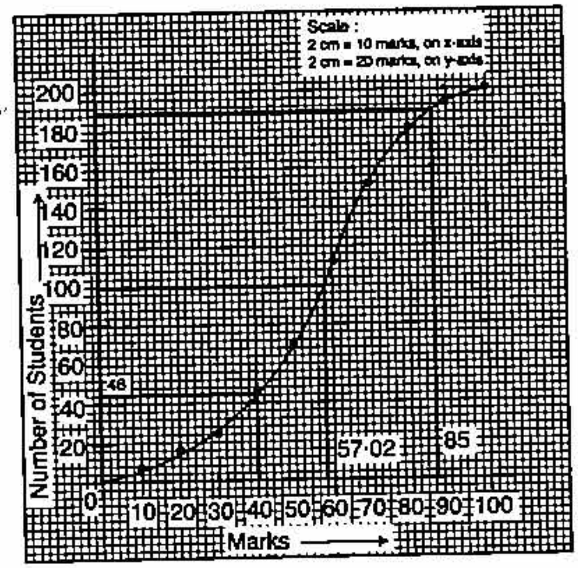
$$= 67{\cdot}5 + \frac{24}{80} \times 5$$

$$= 67{\cdot}5 + 1{\cdot}5 = 69$$ **Ans.**

(ii) Modal class is 55–60 (class with heighest freq.) **Ans.**

(b)

C.I.	f	C.F.
0–10	5	5
10–20	11	16
20–30	10	26
30–40	20	46
40–50	28	74
50–60	37	111
60–70	40	151
70–80	29	180
80–90	14	194
90–100	6	200



(i) $n = 200$

$$\text{Median} = \left(\frac{n}{2}\right)^{\text{th}} \text{observation}$$

$$= \left(\frac{200}{2}\right)^{\text{th}} \text{observation}$$

$$= 100^{\text{th}} \text{ observation} = 57{\cdot}02$$ **Ans.**

(ii) Number of students who failed = 46 **Ans.**

(iii) Number of students who secured grade one = 200 – 1888 = 12. **Ans.**

Question 9.

(a) *Mr. Parekh invested ₹ 52,000 on 100 shares at a discount of ₹ 20 paying 8% dividend. At the end of one year he sells the shares at a premium of ₹ 20. Find*

(i) *The annual dividend.*

(ii) *The profit earned including his dividend.* **[3]**

(b) *Draw a circle of radius 3.5 cm. Mark a point P outside the circle at a distance of 6 cm from the centre. Construct two tangents from P to the given circle. Measure and write down the length of one tangent.* **[3]**

(c) *Prove that* $(\text{cosec } A - \sin A)(\sec A - \cos A)\sec^2 A = \tan A$. **[4]**

Solution :

(a) Investment = ₹ 52,000, N.V = ₹ 100, M.V. of one share = ₹ (100 – 20) = ₹ 80, Dividend = 8%

$$\text{No. of shares} = \frac{\text{Investment}}{\text{MV}}$$

$$= \frac{52{,}000}{80}$$

$$= ₹\ 650$$

(i) $$\text{Annual Dividend} = \frac{8}{100} \times 650 \times 100$$

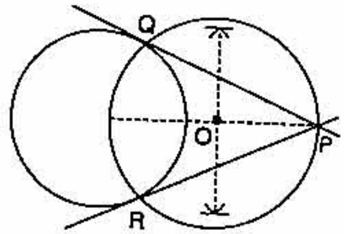
$$= ₹\ 5{,}200$$ **Ans.**

(ii)

$$\begin{aligned} \text{Profit} &= \text{Total S.P.} + \text{Dividend} - \text{Investment} \\ &= 650 \times 120 + 5{,}200 - 52{,}000 \\ &= 78{,}000 + 5{,}200 - 52{,}000 \\ &= ₹\, 31{,}200 \end{aligned}$$

Ans.

(b) Length of the tangent = 4·8 cm.



(c)

$$\begin{aligned} \text{L.H.S.} &= (\text{cosec A} - \sin \text{A})(\sec \text{A} - \cos \text{A}) \cdot \sec^2 \text{A} \\ &= \left(\frac{1}{\sin \text{A}} - \sin \text{A}\right)\left(\frac{1}{\cos \text{A}} - \cos \text{A}\right) \cdot \sec^2 \text{A} \\ &= \left(\frac{1 - \sin^2 \text{A}}{\sin \text{A}}\right)\left(\frac{1 - \cos^2 \text{A}}{\cos \text{A}}\right) \cdot \sec^2 \text{A} \\ &= \frac{\cos^2 \text{A}}{\sin \text{A}} \cdot \frac{\sin^2 \text{A}}{\cos \text{A}} \times \frac{1}{\cos^2 \text{A}} \\ &= \frac{\sin \text{A}}{\cos \text{A}} \\ &= \tan \text{A} = \text{R.H.S.} \end{aligned}$$

Hence Proved

Question 10.

(a) *6 is the mean proportion between two numbers x and y and 48 is the third proportional of x and y. Find the numbers.* **[3]**

(b) *In what period of time will ₹ 12,000 yield ₹ 3,972 as compound interest at 10% per annum, if compounded on an yearly basis ?* **[3]**

(c) *A man observes the angle of elevation of the top of a building to be 30°. He walks towards it in a horizontal line through its base. On covering 60 m the angle of elevation changes to 60°. Find the height of the building correct to the nearest metre.* **[4]**

Solution :

(a)

$$xy = 6^2$$

$\Rightarrow$

$$xy = 36 \qquad \ldots(1)$$

$$x : y :: y : 48$$

$$\frac{x}{y} = \frac{y}{48}$$

$\Rightarrow$

$$y^2 = 48x \qquad \ldots(2)$$

Subtituting the value of x from (1),

$$y^2 = 48 \times \frac{36}{y}$$

$$\Rightarrow \quad y^3 = 48 \times 36$$
$$y^3 = 6 \times 8 \times 6 \times 6$$
$$\therefore \quad y = 12$$
$$\therefore \quad x = 3$$

Ans.

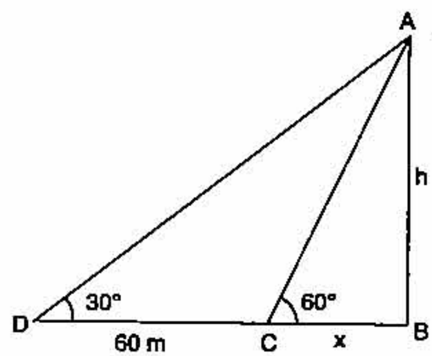
(b) *Given* : P = ₹ 12,000, C.I. = ₹ 3,972, R% = 10% p.a.

Let
$$A = P\left(1 + \frac{R}{100}\right)^n$$
$$\Rightarrow \quad 15{,}972 = 12{,}000\left(1 + \frac{10}{100}\right)^n$$
$$\frac{1331}{1000} = \left(\frac{11}{10}\right)^n$$
$$\therefore \quad \left(\frac{11}{10}\right)^3 = \left(\frac{11}{10}\right)^n$$
$$\Rightarrow \quad n = 3 \text{ years.}$$

Ans.

(c) Let BC = x and AB = h

In right angled Δ ADB



$$\tan 30° = \frac{h}{60 + x}$$
$$\Rightarrow \quad 60 + x = h\sqrt{3}$$

Now right angled Δ ACB

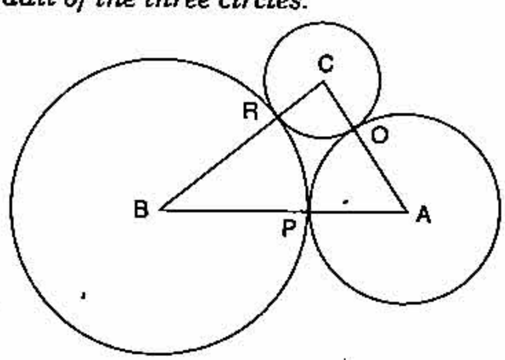
$$\tan 60° = \frac{h}{x}$$
$$\Rightarrow \quad x = \frac{h}{\sqrt{3}}$$

Equating 'x', $$h\sqrt{3} - 60 = \frac{h}{\sqrt{3}}$$
$$\Rightarrow \quad 3h - 60\sqrt{3} = h$$
$$2h = 60\sqrt{3}$$
$$\therefore \quad h = 30\sqrt{3}$$
$$= 51{\cdot}96 \text{ m.}$$

Ans.

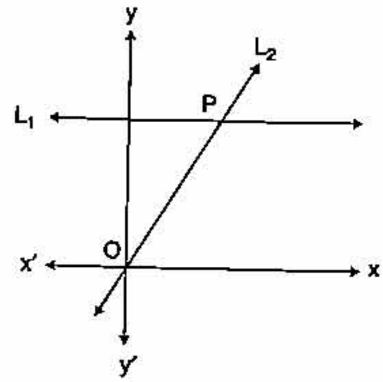
Question 11.

(a) *ABC is a triangle with AB = 10 cm, BC = 8 cm and AC = 6 cm (not drawn to scale). Three circles are drawn touching each other with the vertices as their centres. Find the radii of the three circles.* **[3]**



(b) *₹ 480 is divided equally among 'x' children. If the number of children were 20 more then each would have got ₹ 12 less. Find 'x'.* **[3]**

(c) *Given equation of line L_1 is $y = 4$.*



(i) *Write the slope of line L_2 if L_2 is the bisector of angle O.*

(ii) *Write the co-ordinates of point P.*

(iii) *Find the equation of L_2.* **[4]**

Solution :

(a) Let the three radii be x, y, z respectively.

$$x + y = 10 \quad ...(1)$$
$$y + z = 8 \quad ...(2)$$
$$x + z = 6 \quad ...(3)$$

Adding equation's (1), (2) and (3), $2x + 2y + 2z = 24$

$\therefore$ $x + y + z = 12$ ...(4)

Subtracting each equation (1), (2) and (3) from equation (4), we get

$z = 2$ cm, $x = 4$ cm, $y = 6$ cm. **Ans.**

(b) Initial share of each child $= \dfrac{480}{x}$

New share of each child $= \dfrac{480}{x+20}$

Difference in share is ₹ 12

$$\therefore \quad \frac{480}{x} - \frac{480}{x+20} = 12$$

$$\frac{1}{x} - \frac{1}{x+20} = \frac{12}{480} = \frac{1}{40}$$

$$\frac{x + 20 - x}{x(x+20)} = \frac{1}{40}$$

$$x^2 + 20x = 800$$

$$x^2 + 20x - 800 = 0$$

$$x^2 + 40x - 20x - 800 = 0$$

$$x(x + 40) - 20(x + 40) = 0 \text{ (not possible)}$$

$$x = 20 \text{ or } x = -40$$

$$\therefore \quad x = 20$$ **Ans.**

(c) (i) Slope of L_2 is $m = \tan 45°$

$\therefore$ $m = 1$ (L_2 makes an angle of 45° with X axis)

(ii) Equation of line L_2

$y - 0 = 1(x - 0)$ It passes through (0, 0)

$\therefore$ Equation of L_2 is $y = x$

P can be obtained by solving L_1 and L_2 simultaneously,

L_1 $y = 3$

L_2 $y = x$

On solving, we get $x = 3, y = 3$

Co-ordinate of P (3, 3) **Ans.**

(iii) Equation of L_2 is $y = x$ [as solved above part (ii)]. **Ans.**