ICSE Paper 2010

MATHEMATICS

SECTION A [40 Marks]

*(Answer **all** questions from this Section.)*

Question 1.

(a) *Solve the following inequation and represent the solution set on the number line.*

$$-3 < -\frac{1}{2} - \frac{2x}{3} \leq \frac{5}{6}, x \in R$$

[3]

(b) *Tarun bought an article for ₹ 8,000 and spent ₹ 1,000 for transportation. He marked the article at ₹ 11,700 and sold it to a customer. If the customer had to pay 10% sales tax, find*

(i) *The customer's price.*

(ii) *Tarun's profit percent.*

[3]

(c) *Mr. Gupta opened a recurring deposit account in a bank. He deposited ₹ 2,500 per month for two years. At the time of maturity he got ₹ 67,500. Find :*

(i) *the total interest earned by Mr. Gupta.*

(ii) *the rate of interest per annum.*

[4]

Solution :

(a) *Given* : $-3 < -\frac{1}{2} - \frac{2x}{3} \leq \frac{5}{6}, x \in \text{R}$

$$-3 < -\frac{1}{2} - \frac{2x}{3} \quad \text{and} \quad -\frac{1}{2} - \frac{2x}{3} \leq \frac{5}{6}$$

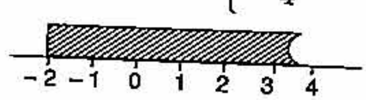
$$-3 + \frac{1}{2} < -\frac{2x}{3} \quad \text{and} \quad -\frac{2x}{3} \leq \frac{5}{6} + \frac{1}{2}$$

$$-\frac{5}{2} < -\frac{2x}{3} \quad \text{and} \quad -\frac{2x}{3} \leq \frac{4}{3}$$

$$\frac{5}{2} > \frac{2x}{3} \quad \text{and} \quad -x \leq 2$$

$$x < \frac{15}{4} \quad \text{and} \quad x \geq -2$$

$$\text{Solution set} = \left\{x : \frac{15}{4} > x \geq -2\right\}$$



(b) *Given* : C.P. = ₹ 8,000 + ₹ 1,000 = ₹ 9,000, M.P. = ₹ 11,700, S.T. = 10%.

(i) Amount to be paid = M.P. + S.T. % of M.P.

$$= 11{,}700 + \frac{10}{100} \times 11{,}700$$

$$= ₹\ 12{,}870$$

Ans.

(ii)

$$\text{Profit} = \text{M.P.} - \text{C.P.} = 11{,}700 - 9{,}000$$
$$= ₹\,2{,}700.$$
$$\text{Profit percent} = \frac{\text{Profit}}{\text{C.P.}} \times 100$$
$$= \frac{2{,}700}{9{,}000} \times 100$$
$$= 30\%.$$

Ans.

(c)

$$\text{Total amount deposited} = ₹\,(2{,}500 \times 24) = ₹\,60{,}000$$
$$\text{Equivalent principal for one month} = ₹\,2{,}500 \times \frac{24\,(24+1)}{2} = ₹\,(62{,}500 \times 12)$$

(i)

$$\text{Total interest} = 67{,}500 - 60{,}000$$
$$= ₹\,7{,}500$$

(ii) Interest on ₹ (62,500 × 12) for 1 month

$$= ₹\left(62{,}500 \times 12 \times \frac{R}{100} \times \frac{1}{12}\right) \qquad \left[\because I = \frac{P \times R \times T}{100}\right]$$
$$7{,}500 = 625\,R.$$
$$\Rightarrow \quad R = 12\%.$$

Ans.

Question 2.

(a) *Given* $A = \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} -4 \\ 5 \end{bmatrix}$ *and* $D = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

Find $AB + 2C - 4D$. **[3]**

(b) *Nikita invests ₹ 6,000 for two years at a certain rate of interest compounded annually. At the end of the first year it amounts to ₹ 6,720. Calculate :*

(i) *the rate of interest.*

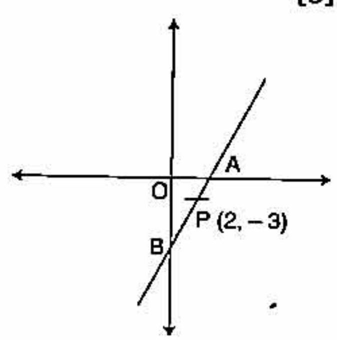
(ii) *the amount at the end of the second year.* **[3]**

(c) *A and B are two points on the x-axis and y-axis respectively. P (2, –3) is the mid point of AB. Find the*

(i) *Coordinates of A and B.*

(ii) *Slope of line AB.*

(iii) *Equation of line AB.* **[4]**



Solution :

(a) *Given* : $A = \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}, B = \begin{bmatrix} 6 \\ 1 \end{bmatrix}, C = \begin{bmatrix} -4 \\ 5 \end{bmatrix}, D = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

$$AB + 2C - 4D = \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 6 \\ 1 \end{bmatrix} + 2\begin{bmatrix} -4 \\ 5 \end{bmatrix} - 4\begin{bmatrix} 2 \\ 2 \end{bmatrix}$$
$$= \begin{bmatrix} 16 \\ -2 \end{bmatrix} + \begin{bmatrix} -8 \\ 10 \end{bmatrix} - \begin{bmatrix} 8 \\ 8 \end{bmatrix}$$
$$= \begin{bmatrix} 0 \\ 0 \end{bmatrix} = 0$$

Ans.

(b) (i) *Given* : Principal = ₹ 6,000, Time = 2 year, After one year amount = ₹ 6,720.

For Ist year :
$$P + I = ₹\,6,720$$
$$6,000 + \frac{P \times R \times 1}{100} = 6,720$$
$$\frac{6,000 \times R}{100} = 720$$
$$\Rightarrow \quad R = 12\%$$
Ans.

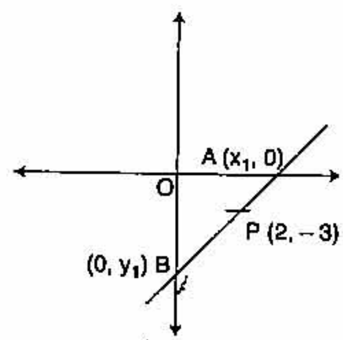
(ii) $\because \quad A = P\left(1 + \frac{r}{100}\right)^n$

Amount at the end of 2nd year
$$= 6,000\left(1 + \frac{12}{100}\right)^2$$
$$= 6,000\left(1 + \frac{3}{25}\right)^2$$
$$= 6,000\left(\frac{28}{25} \times \frac{28}{25}\right) = \frac{37,632}{5}$$
$$= ₹\,7,526{\cdot}40.$$
Ans.

(c) *Given* : A $(x_1, 0)$, B $(0, y_1)$

(i) Mid point coordinates
$$\frac{x_1 + 0}{2} = 2 \quad \Rightarrow \quad x_1 = 4$$
$$\frac{0 + y_1}{2} = -3 \quad \Rightarrow \quad y_1 = -6$$

Coordinates of A (4, 0) and B (0, –6) **Ans.**



(ii) Slope of line AB
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{-6 - 0}{0 - 4} = \frac{-6}{-4} = \frac{3}{2}$$
Ans.

(iii) Equation of line
$$y - y_1 = m(x - x_1)$$
$$y - 0 = \frac{3}{2}(x - 4)$$
$$\Rightarrow \quad 2y = 3x - 12$$
$$\Rightarrow \quad 3x - 2y - 12 = 0$$
Ans.

Question 3.

(a) *Cards marked with numbers 1, 2, 3, 4 ... 20 are well shuffled and a card is drawn at random. What is the probability that the number of the cards is*

(i) *a prime number*

(ii) *divisible by 3*

(iii) *a perfect square ?* **[3]**

(b) *Without using trigonometric tables evaluate :*
$$\frac{\sin 35^\circ \cos 55^\circ + \cos 35^\circ \sin 55^\circ}{\operatorname{cosec}^2 10^\circ - \tan^2 80^\circ}$$
[3]

(c) *(Use graph paper for this question)*

A(0, 3), B(3, –2) and O(0, 0) are the vertices of triangle ABO.

(i) *Plot the triangle on a graph sheet taking 2 cm = 1 unit on both the axes.*

(ii) *Plot D the reflection of B in the Y axis, and write its co-ordinates.*

(iii) *Give the geometrical name of the figure ABOD.*

(iv) *Write the equation of the line of symmetry of the figure ABOD.* **[4]**

Solution :

(a) *Given* : Cards marked with numbers 1, 2, ... 20.

$n(S) = 20$

(i) Prime Numbers = 2, 3, 5, 7, 11, 13, 17, 19

$n(E) = 8$

$$P\text{ (Prime number)} = P(A) = \frac{n(E)}{n(S)} = \frac{8}{20} = \frac{2}{5}$$ **Ans.**

(ii) No. divided by 3 = 3, 6, 9, 12, 15, 18

$n(E) = 6$

$$P\text{ (no. divided by 3)} = P(A) = \frac{n(E)}{n(S)} = \frac{6}{20} = \frac{3}{10}$$ **Ans.**

(iii) No. perfect square = 1, 4, 9, 16

$n(E) = 4$

$$P\text{ (Perfect square)} = P(A) = \frac{n(E)}{n(S)} = \frac{4}{20} = \frac{1}{5}$$ **Ans.**

(b) *Given* :

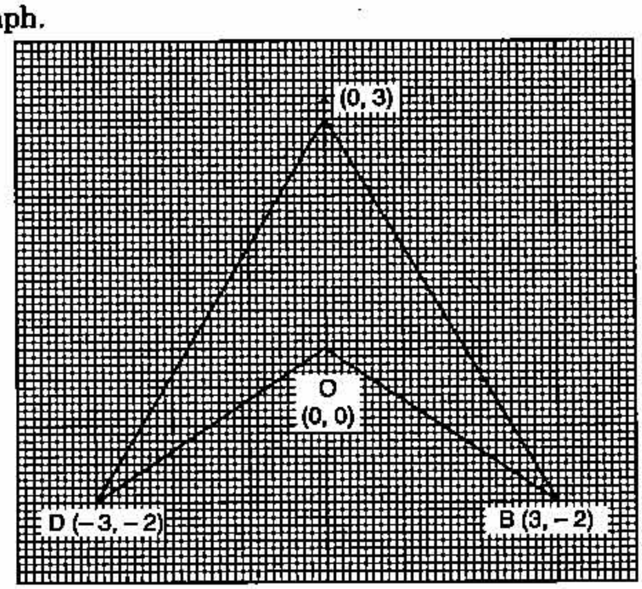
$$\frac{\sin 35° \cos 55° + \cos 35° \sin 55°}{\text{cosec}^2 10° - \tan^2 80°}$$

$$= \frac{\sin (90-55)° \cos 55° + \cos (90-55)° \sin 55°}{\text{cosec}^2 10° - \tan^2 (90-10)°}$$

$$= \frac{\cos 55° \cos 55° + \sin 55° \sin 55°}{(1 + \cot^2 10°) - \cot^2 10°}$$

$$= \frac{\cos^2 55° + \sin^2 55°}{1 + \cot^2 10° - \cot^2 10°} = \frac{1}{1} = 1$$ **Ans.**

(c) (i) See graph.



(ii) Coordinate of D = (–3, –2)

(iii) Geometrical name of ABOD is arrow.

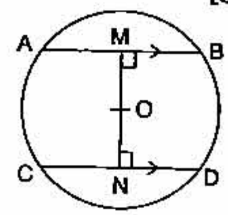
(iv) Equation of the line of symmetry is

$$x = 0$$

Question 4.

(a) *When divided by $x - 3$ the polynomials $x^3 - px^2 + x + 6$ and $2x^3 - x^2 - (p + 3)x - 6$ leave the same remainder. Find the value of 'p'.* **[3]**

(b) *In the figure given alongside AB and CD are two parallel chords and O is the centre. If the radius of the circle is 15 cm, find the distance MN between the two chords of length 24 cm and 18 cm respectively.* **[3]**



(c) *The distribution given below shows the marks obtained by 25 students in an aptitude test. Find the mean, median and mode of the distribution.* **[4]**

Marks obtained	*5*	*6*	*7*	*8*	*9*	*10*
No. of students	*3*	*9*	*6*	*4*	*2*	*1*

Solution :

(a) *Given* :

$$f(x) = x^3 - px^2 + x + 6$$
$$g(x) = 2x^3 - x^2 - (p+3)x - 6$$

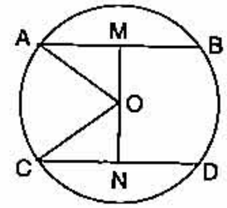
when $f(x)$ is divided by $(x - 3)$ remainder $f(3)$ and $f(x)$ is divided by $(x - 3)$ remainder $g(3)$.

$$f(3) = g(3)$$
$$(3)^3 - (3)^2 p + 3 + 6 = 2(3)^3 - (3)^2 - (p+3)3 - 6$$
$$27 - 9p + 3 + 6 = 54 - 9 - (p+3)3 - 6$$
$$\Rightarrow \quad 36 - 9p = 30 - 3p$$
$$\Rightarrow \quad 9p - 3p = 36 - 30$$
$$\Rightarrow \quad 6p = 6$$
$$\Rightarrow \quad p = 1$$

Ans.

(b) *Given* : OA = OC = 15 cm, AB = 24 cm, CD = 18 cm.

Now AM = 12, CN = 9



In Δ OAM,

$$OA^2 = OM^2 + AM^2$$
$$OM^2 = OA^2 - AM^2$$
$$= 15^2 - 12^2$$
$$= 225 - 144$$
$$= 81$$
$$OM = 9$$

Similarly, in Δ OCN,

$$OC^2 = ON^2 + CN^2$$
$$ON^2 = OC^2 - CN^2 = 15^2 - 9^2$$
$$= 225 - 81 = 144$$
$$ON = 12$$
$$MN = OM + ON = 9 + 12 = 21 \text{ cm.}$$

Ans.

(c)

x_i	f_i	$x_i f_i$	cf
5	3	15	3
6	9	54	12
7	6	42	18
8	4	32	22
9	2	18	24
10	1	10	25
	$\Sigma f = 25$	$\Sigma x_i f_i = 171$	

$$\text{Mean} = \frac{\Sigma x_i f_i}{N} = \frac{171}{25} = 6 \cdot 84$$ **Ans.**

$\because$ $n = 25$ (odd)

$$\text{Median} = \left(\frac{n+1}{2}\right)^{\text{th}} \text{ term} = 13^{\text{th}} \text{ term} = 7$$ **Ans.**

$$\text{Mode} = 6 \text{ (maximum freq.)}$$ **Ans.**

SECTION B [40 Marks]

Answer any ***four*** *Questions in this Section.*

Question 5.

(a) *Without solving the following quadratic equation, find the value of 'p' for which the roots are equal.*

$$px^2 - 4x + 3 = 0$$ **[3]**

(b) *Rohit borrows ₹ 86,000 from Arun for two years at 5% per annum simple interest. He immediately lends out this money to Akshay at 5% compound interest compounded annually for the same period. Calculate Rohit's profit in the transaction at the end of the two years.* **[3]**

(c) *Mrs. Kapoor opened a Saving Bank Account in State Bank of India on 9th January 2008. Her pass book entries for the year 2008 are given below :*

Date	*Particulars*	*Withdrawals (in ₹)*	*Deposits (in ₹)*	*Balance (in ₹)*
Jan 9, 2008	*By Cash*	—	*10,000*	*10,000*
Feb 12, 2008	*By Cash*	—	*15,500*	*25,500*
April 6, 2008	*To Cheque*	*3,500*	—	*22,000*
April 30, 2008	*To Self*	*2,000*	—	*20,000*
July 16, 2008	*By Cheque*	—	*6,500*	*26,500*
Aug. 4, 2008	*To Self*	*5,500*	—	*21,000*
Aug. 20, 2008	*To Cheque*	*1,200*	—	*19,800*
Dec. 12, 2008	*By Cash*	—	*1,700*	*21,500*

Mrs. Kapoor closed the account on 31st December, 2008. If the bank pays interest at 4% per annum, find the interest Mrs. Kapoor receives on closing the account. Give your answer correct to the nearest rupee. **[4]**

Solution :

(a) Roots are equal $\Rightarrow$

$$b^2 - 4ac = 0$$

$$b^2 = 4ac$$

Given : $a = p, b = 4, c = 3.$

$$\Rightarrow \quad 16 = 4.p.3$$

$$\Rightarrow \quad p = \frac{16}{12} = \frac{4}{3}$$ **Ans.**

(b) *Given* : P = 86,000, R = 5%, T = 2 years.

$$\text{S.I.} = \frac{P \times R \times T}{100} = \frac{86,000 \times 5 \times 2}{100} = ₹\,8,600$$

$$\text{C.I.} = P\left[\left(1 + \frac{R}{100}\right)^T - 1\right]$$

$$= 86,000\left[\left(1 + \frac{5}{100}\right)^2 - 1\right] = 86,000\left[\left(\frac{21}{20}\right)^2 - 1\right] = 86,000 \times \frac{41}{400} = ₹\,8,815$$

$$\text{Profit} = \text{C.I.} - \text{S.I.} = 8,815 - 8,600$$
$$= ₹\,215$$ **Ans.**

(c) Minimum balance for the month

January	–	10,000
February	–	10,000
March	–	25,500
April	–	20,000
May	–	20,000
June	–	20,000
July	–	20,000
August	–	19,800
September	–	19,800
October	–	19,800
November	–	19,800

$$\text{Principal} = ₹\,2,04,700,\ R = 4\%$$

$$\text{S.I.} = \frac{P \times R \times T}{100} = \frac{2,04,700 \times 4 \times 1}{100 \times 12}$$

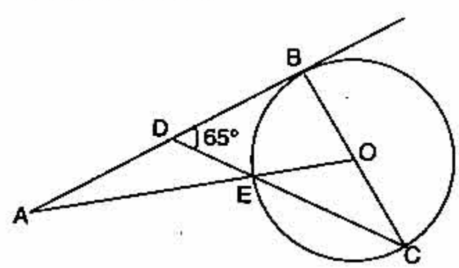
$$= ₹\,682{\cdot}33 = ₹\,682.$$ **Ans.**

Question 6.

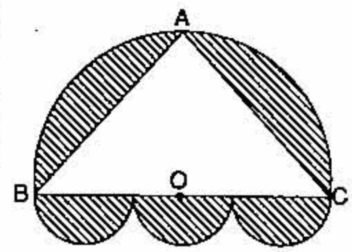
(a) *A manufacturer marks an article for ₹ 5,000. He sells it to a wholesaler at a discount of 25% on the market price and the wholesaler sells it to a retailer at a discount of 15% on the market price. The retailers sells it to a consumer at the market price and at each stage the VAT is 8%. Calculate the amount of VAT received by the Government from :*

(i) *the wholesaler* (ii) *the retailer.* **[3]**

(b) *In the following figure O is the centre of the circle and AB is a tangent to it at point B. ∠BDC = 65°. Find ∠BAO.* **[3]**



(c) *A doorway is decorated as shown in the figure. There are four semi-circles. BC, the diameter of the larger semi circle is of length 84 cm. Centres of the three equal semi-circles lie on BC. ABC is an isosceles triangle with AB = AC. If BO = OC, find the area of the shaded region.* $\left(\text{Take } \pi = \frac{22}{7}\right)$ **[4]**



Solution :

(a) *Given :*

$$\text{Cost of manufacturer} = ₹\,5{,}000$$

$$\begin{aligned}\text{S.P. of manufacturer} &= \text{C.P. of wholesaler}\\ &= 5{,}000 - \frac{25}{100} \times 5{,}000\\ &= 5{,}000 - 1{,}250\\ &= ₹\,3{,}750\end{aligned}$$

$$\begin{aligned}\text{S.P. of wholesaler} &= \text{C.P. of retailer}\\ &= 5{,}000 - \frac{15}{100} \times 5{,}000\\ &= 5{,}000 - 750\\ &= ₹\,4{,}250\end{aligned}$$

$$\begin{aligned}\text{S.P. of retailer} &= \text{C.P. of consumer}\\ &= ₹\,5{,}000\end{aligned}$$

(i) $$\begin{aligned}\text{VAT by the wholesaler} &= \frac{8}{100} \times 3{,}750\\ &= ₹\,300\end{aligned}$$ **Ans.**

(ii) $$\begin{aligned}\text{VAT by retailer} &= \frac{8}{100} \times (4{,}250 - 3{,}750)\\ &= \frac{8}{100} \times 500\\ &= ₹\,40.\end{aligned}$$ **Ans.**

(b) AB is tangent $\Rightarrow \angle ABO = 90°$

$$\angle BDC = 65° \text{ (given)}$$

$$\Rightarrow \quad \angle BCD = 90° - 65° = 25°$$

$$\begin{aligned}\angle BOE &= 2 \times 25° \quad \text{(angle at centre)}\\ &= 50°\end{aligned}$$

$$\begin{aligned}\angle BAO &= 90° - \angle BOE\\ \angle BAO &= 90° - 50°\\ &= 40°\end{aligned}$$ **Ans.**

(c) Let AB = AC = x cm.

As angle in semi circle is 90°

i.e., $\angle A = 90°$

In right angled Δ ABC, by Pythagoras theorem, we get

$$AB^2 + AC^2 = BC^2$$

$$x^2 + x^2 = 84^2$$

$$2x^2 = 84 \times 84$$

$\therefore$ $$x^2 = 84 \times 42$$

Now $$\text{Area of } \Delta ABC = \frac{1}{2} \times AB \times AC$$
$$= \frac{1}{2} \times 84 \times 42$$
$$= 1764 \text{ cm}^2.$$

$$\text{Diameter of semicircle } (2r) = 84 \text{ cm}$$
$$\text{Radius } (r) = \frac{1}{2} \times 84 = 42 \text{ cm}$$

$\therefore$ $$\text{Area of semicircle} = \frac{1}{2}\pi r^2 = \frac{1}{2} \times \frac{22}{7} \times 42 \times 42$$
$$= 2772 \text{ cm}^2.$$

$$\text{Diameter of each (three equal) semicircles} = \frac{1}{3} \times 84 = 28 \text{ cm}.$$
$$\text{Radius of the 3 equal semicircles} = \frac{1}{2} \times 28 = 14 \text{ cm}.$$

$\therefore$ $$\text{Area of three equal semi circles} = 3 \times \frac{1}{2}\pi r^2$$
$$= 3 \times \frac{1}{2} \times \frac{22}{7} \times 14 \times 14$$
$$= 924 \text{ cm}^2.$$

Area of shaded region = Area of semicircles + Area of three equal circles − Area of Δ ABC

$$= 2772 + 924 - 1764$$
$$= 3696 - 1764$$
$$= 1932 \text{ cm}^2.$$ **Ans.**

Question 7.

(a) *Use ruler and compasses only for this question :*

(i) *Construct ΔABC, where $AB = 3.5$ cm, $BC = 6$ cm and $\angle ABC = 60°$.*

(ii) *Construct the locus of points inside the triangle which are equidistant from BA and BC.*

(iii) *Construct the locus of points inside the triangle which are equidistant from B and C.*

(iv) *Mark the point P which is equidistant from AB, BC and also equidistant from B and C. Measure and record the length of PB.* **[3]**

(b) *The equation of a line is $3x + 4y - 7 = 0$. Find*

(i) *the slope of the line.*

(ii) *the equation of a line perpendicular to the given line and passing through the intersection of the lines $x - y + 2 = 0$ and $3x + y - 10 = 0$.* **[3]**

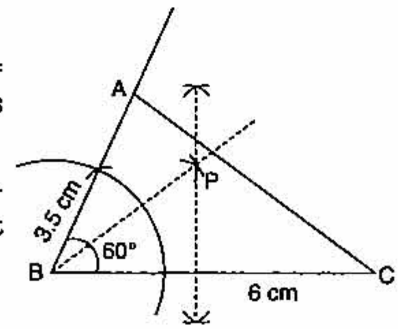
(c) *The mean of the following distribution is 52 and the frequency of class interval 30-40 is 'f'. Find 'f'.*

Class Interval	*10-20*	*20-30*	*30-40*	*40-50*	*50-60*	*60-70*	*70-80*
Frequency	5	3	*f*	7	2	6	13

[4]

Solution :

(a) Steps of Construction :



(i) Draw BC = 6 cm and make an angle at B = 60°. Cut BA = 3·5 cm and meet A to C. This is the required Δ ABC.

(ii) Draw the bisector of Δ ABC and perpendicular bisector of BC; both intersecting at P.

(iii) P is the required point. PB = 3·5 cm.

(b) *Given* : Equation of the line is

$$3x + 4y - 7 = 0$$

$$4y = -3x + 7$$

$$y = -\frac{3}{4}x + \frac{7}{4}$$

(i) Slope of the line $(m_1) = -\frac{3}{4}$ **Ans.**

(ii) Slope of the perpendicular line $(m_2) = \frac{-1}{m_1} = \frac{-1}{-3/4} = \frac{4}{3}$

Intersection of the lines $x - y + 2 = 0$ ...(i)

and $3x + y - 10 = 0$ ...(ii)

By Adding equation (i) and (ii) $4x = 8 \Rightarrow x = 2$

Put $x = 2$, in equation (i) we get

$$2 - y + 2 = 0 \Rightarrow y = 4$$

Equation of line $y - y_1 = m_2(x - x_1)$

$$y - 4 = \frac{4}{3}(x - 2)$$

$\Rightarrow$ $4x - 3y + 4 = 0$ **Ans.**

(c)

Interval	*Frequency* (f_i)	x_i	$d_i = x_i - A$	$f_i d_i$
10–20	5	15	– 30	– 150
20–30	3	25	– 20	– 60
30–40	f	35	– 10	– 10f
40–50	7	45 A	0	0
50–60	2	55	10	20
60–70	6	65	20	120
70–80	13	75	30	390
	36 + f			$\Sigma f_i d_i = 320 - 10f$

$$\text{Mean} = A + \frac{\Sigma f_i d_i}{N}$$

$$52 = 45 + \frac{320 - 10f}{36 + f}$$

$$\Rightarrow \quad 7 = \frac{320 - 10f}{36 + f}$$

$$\Rightarrow \quad 252 + 7f = 320 - 10f$$

$$\Rightarrow \quad 17f = 68$$

$$\Rightarrow \quad f = 4$$

Ans.

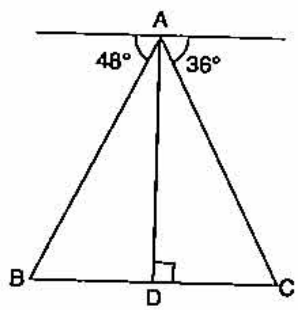
Question 8.

(a) *Use the Remainder Theorem to factorise the following expression :*

$$2x^3 + x^2 - 13x + 6$$ **[3]**

(b) *If x, y, z are in continued proportion, prove that* $\frac{(x+y)^2}{(y+z)^2} = \frac{x}{z}$. **[3]**

(c) *From the top of a light house 100 m high the angles of depression of two ships on opposite sides of it are 48° and 36° respectively. Find the distance between the two ships to the nearest metre.* **[4]**



Solution :

(a) *Given* :

$$f(x) = 2x^3 + x^2 - 13x + 6$$
$$f(1) = 2 + 1 - 13 + 6 \neq 0$$
$$f(-1) = -2 + 1 + 13 + 6 \neq 0$$
$$f(2) = 16 + 4 - 26 + 6 = 0$$

So, $x - 2$ is one factor of $f(x)$ by remainder theorem.

$$\begin{array}{r|l} & 2x^2 + 5x - 3 \\ x - 2 & 2x^3 + x^2 - 13x + 6 \\ & \underline{2x^3 - 4x^2} \\ & \quad 5x^2 - 13x + 6 \\ & \quad \underline{5x^2 - 10x} \\ & \qquad -3x + 6 \\ & \qquad \underline{-3x + 6} \\ & \qquad \times \end{array}$$

$\therefore$ The other factors of $f(x)$ are the factors of $2x^2 + 5x - 3$.

$$= 2x^2 + 6x - x - 3$$
$$= 2x(x + 3) - 1(x + 3)$$
$$= (2x - 1)(x + 3)$$

Hence, $2x^3 + x^2 - 13x + 6 = (2x - 1)(x + 3)(x - 2)$ **Ans.**

(b) If x, y, z are in continued proportion

$$\frac{x}{y} = \frac{y}{z} = k$$

$\Rightarrow$ $y = kz$

and $x = xy = k^2z$

$$\text{L.H.S.} = \frac{(x+y)^2}{(y+z)^2} = \frac{(k^2z + kz)^2}{(kz+z)^2}$$

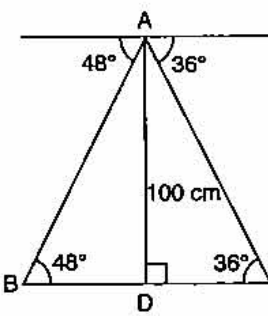
$$= \frac{k^2z^2(k+1)^2}{z^2(k+1)^2}$$

$$= k^2$$

$$\text{R.H.S.} = \frac{x}{z} = \frac{k^2z}{z} = k^2$$

Hence L.H.S. = R.H.S. **Proved**

(c) In Δ ABD, $\tan 48° = \frac{AD}{BD}$



$\Rightarrow$ $1{\cdot}11 = \frac{100}{BD}$

$$BD = \frac{100}{1{\cdot}11} = 90{\cdot}09 \text{ m}$$

In Δ ACD, $\tan 36° = \frac{AD}{DC}$

$\Rightarrow$ $0{\cdot}7265 = \frac{100}{DC}$

$\Rightarrow$ $DC = \frac{100}{0{\cdot}7265} = 137{\cdot}64 \text{ m}$

$\therefore$ $BC = BD + DC$

$= 90{\cdot}09 + 137{\cdot}64$

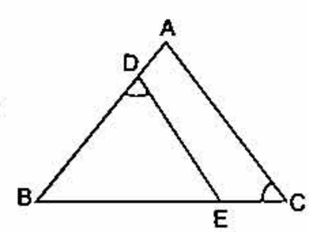
$= 227{\cdot}73$ m. **Ans.**

Question 9.

(a) *Evaluate* :

$$\begin{bmatrix} 4\sin 30° & 2\cos 60° \\ \sin 90° & 2\cos 0° \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$$ **[3]**

(b) *In the figure ABC is a triangle with $\angle EDB = \angle ACB$.*
Prove that $\Delta ABC \sim \Delta EBD$.
If BE = 6 cm, EC = 4 cm, BD = 5 cm and area of Δ BED = 9 cm^2. Calculate the :
(i) *length of AB*
(ii) *area of Δ ABC.* **[4]**



(c) *Vivek invests ₹ 4,500 in 8%, ₹ 10 shares at ₹ 15. He sells the shares when the price rises to ₹ 30, and invests the proceeds in 12% ₹ 100 shares at ₹ 125. Calculate :*
(i) *the sale proceeds.*
(ii) *the number of ₹ 125 shares he buys.*
(iii) *the change in his annual income from dividend.* **[4]**

Solution :

(a) *Given :*

$$\begin{bmatrix} 4\sin 30^\circ & 2\cos 60^\circ \\ \sin 90^\circ & 2\cos 0^\circ \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 4\times\frac{1}{2} & 2\times\frac{1}{2} \\ 1 & 2\times 1 \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 13 & 14 \\ 14 & 13 \end{bmatrix}$$ **Ans.**

(b)

$\angle EDB = \angle ACB$ (given)

$\angle DBE = \angle ABC$

$\angle DEB = \angle BAC$ (AA axiom)

$\Rightarrow \Delta ABC \sim \Delta EBD$ **Proved**

(i) *Given* : BE = 6 cm, EC = 4 cm, BD = 5 cm.

$$\frac{AB}{EB} = \frac{BC}{BD} = \frac{AC}{ED}$$

$$\frac{AB}{EB} = \frac{BC}{BD}$$

$$\Rightarrow \frac{AB}{6} = \frac{BE+EC}{5} = \frac{6+4}{5} = \frac{10}{5} = 2$$

$\Rightarrow$ AB = 12 cm **Ans.**

(ii)

$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta EBD} = \frac{AB^2}{EB^2} = \frac{144}{36}$$

$$\frac{\text{Area of } \Delta ABC}{9} = \frac{(12)^2}{(6)^2}$$

$$\Rightarrow \text{Area of } \Delta ABC = \frac{144\times 9}{36} = 36 \text{ m.}$$ **Ans.**

(c)

$$\text{Number of shares bought} = \frac{4{,}500}{15} = 300$$

Total face value = ₹ 300 × 10 = ₹ 3,000

$$\text{Dividend} = \frac{8}{100}\times 3{,}000 = ₹\ 240.$$

Amount received on selling 300 shares for ₹ = 300 × 30 = ₹ 9,000

(i) Sale proceeds = ₹ 9,000 – ₹ 4,500 = ₹ 4,500 **Ans.**

(ii) Number of shares bought at ₹ 125 $= \frac{9{,}000}{125} = 72$ **Ans.**

(iii)

$$\text{Total face value of 72 shares} = ₹\, 72 \times 100$$
$$= ₹\, 7{,}200$$
$$\text{Dividend} = \frac{12}{100} \times 7{,}200$$
$$= ₹\, 864.$$
$$\text{Change in his annual income} = 864 - 240$$
$$= ₹\, 624.$$

Ans.

Question 10.

(a) *A positive number is divided into two parts such that the sum of the squares of the two parts is 208. The square of the larger part is 8 times the smaller part. Taking x as the smaller part of the two parts, find the number.* **[4]**

(b) *The monthly income of a group of 320 employees in a company is given below :*

Monthly Income	*No. of Employees*
6000–7000	20
7000–8000	45
8000–9000	65
9000–10000	95
10000–11000	60
11000–12000	30
12000–13000	5

Draw an ogive of the given distribution on a graph sheet taking 2 cm = ₹ 1,000 on one axis and 2 cm = 50 employees on the other axis. From the graph determine :

(i) *the median wage.*

(ii) *the number of employees whose income is below ₹ 8,500*

(iii) *If the salary of a senior employee is above ₹ 11,500, find the number of senior employees in the company.*

(iv) *the upper quartile.* **[6]**

Solution :

(a) Let x and y are the two parts.

$$x^2 + y^2 = 208 \quad \text{...(1)}$$
$$y^2 = 8x \quad \text{...(2)}$$
$$\Rightarrow \quad x^2 + 8x - 208 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Here, $a = 1$, $b = 8$, $c = -208$

$$= \frac{-8 \pm \sqrt{(8)^2 - 4 \times 1 \times (-208)}}{2 \times 1}$$

$$= \frac{-8 \pm \sqrt{64 + 832}}{2}$$

$$= \frac{-8 \pm 29 \cdot 93}{2} = \frac{-8 + 29 \cdot 93}{2} \text{ or } \frac{-8 - 29 \cdot 93}{2}$$

$$= -18 \cdot 96 \text{ or } 10 \cdot 97$$

$$y^2 = 8x$$

$$= 8 \times 10 \cdot 97$$

$$= 87 \cdot 76$$

$$y = 9 \cdot 37$$

$$\text{Number} = x + y$$

$$= 10 \cdot 97 + 9 \cdot 37$$

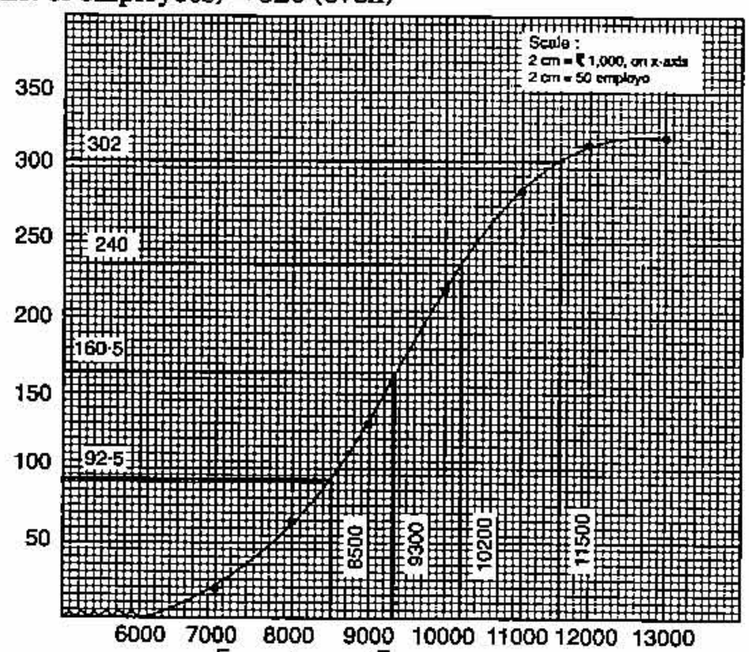
$$= 20 \cdot 34$$

Ans.

(b)

Monthly Income	*No. of Employees*	*C.F.*
6000–7000	20	20
7000–8000	45	65
8000–9000	65	130
9000–10000	95	225
10000–11000	60	285
11000–12000	30	315
12000–13000	5	320
	320	

Here n (no. of employees) = 320 (even)



(i) $$\text{Median} = \frac{1}{2}\left[\frac{n}{2} + \left(\frac{n}{2} + 1\right)\right] = \frac{1}{2}[160 + 161] = 160 \cdot 5$$

Required median = ₹ 9,300 (from graph) **Ans.**

(ii) Number of employees whose income is below ₹ 8,500 = 92·5 approx. **Ans.**

(iii) Number of senior employees in the company = 320 − 302 = 18. **Ans.**

(iv) $$\text{Upper quartile} = \frac{3n}{4} = \frac{3 \times 320}{4} = 240$$

$$\text{Upper quartile} = 10{,}200.$$ **Ans.**

Question 11.

(a) *Construct a regular hexagon of side 4 cm. Construct a circle circumscribing the hexagon.* **[3]**

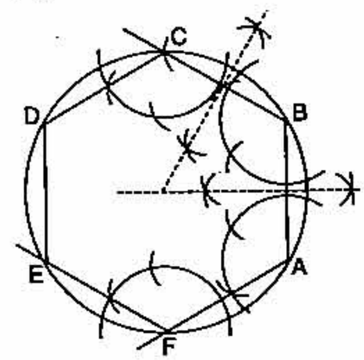
(b) *A hemispherical bowl of diameter 7.2 cm is filled completely with chocolate sauce. This sauce is poured into an inverted cone of radius 4.8 cm. Find the height of the cone.* **[3]**

(c) *Given* $x = \dfrac{\sqrt{a^2+b^2} + \sqrt{a^2-b^2}}{\sqrt{a^2+b^2} - \sqrt{a^2-b^2}}$

Use componendo and dividendo to prove that $b^2 = \dfrac{2a^2x}{x^2+1}$. **[4]**

Solution :

(a) Steps of Construction :



(i) Using the given data, construct the regular hexagon ABCDEF with each side equal to 4 cm.

(ii) Draw the perpendicular bisectors of sides AB and AF which intersect each other at point O.

(iii) With O as centre and OA as radius draw a circle which will pass through all the vertices of the regular hexagon ABCDEF.

(b) *Given :* Diameter of hemispherical bowl = 7·2 cm

Radius of hemispherical bowl = 3·6 cm

$$\text{Volume of hemispherical bowl} = \frac{2}{3} \times \pi r^3$$

$$= \frac{2}{3} \times \frac{22}{7} \times 3{\cdot}6 \times 3{\cdot}6 \times 3{\cdot}6$$

$$= 97{\cdot}76 \text{ cm}^3.$$

$$\text{Volume of cone} = \frac{1}{3} \times \pi R^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times 4{\cdot}8 \times 4{\cdot}8 \times h$$

$$= 24{\cdot}14 \times h \text{ cm}^3$$

Volume of cone = Volume of hemisperical bowl

$$\Rightarrow \quad 24{\cdot}14 \times h = 97{\cdot}76$$

$$\Rightarrow \quad h = \frac{97{\cdot}76}{24{\cdot}14}$$

$$= 4{\cdot}05 \text{ cm}.$$ **Ans.**

(c) *Given* : $x = \dfrac{\sqrt{a^2+b^2} + \sqrt{a^2-b^2}}{\sqrt{a^2+b^2} - \sqrt{a^2-b^2}}$

Componendo and dividendo

$$\frac{x+1}{x-1} = \frac{(\sqrt{a^2+b^2} + \sqrt{a^2-b^2}) + (\sqrt{a^2+b^2} - \sqrt{a^2-b^2})}{(\sqrt{a^2+b^2} + \sqrt{a^2-b^2}) - (\sqrt{a^2+b^2} - \sqrt{a^2-b^2})}$$

$$= \frac{2(\sqrt{a^2+b^2})}{2\sqrt{a^2-b^2}}$$

$$\Rightarrow \quad \frac{(x+1)^2}{(x-1)^2} = \frac{a^2+b^2}{a^2-b^2}$$

Again componendo and dividendo

$$\Rightarrow \quad \frac{(x+1)^2+(x-1)^2}{(x+1)^2-(x-1)^2} = \frac{a^2+b^2+a^2-b^2}{a^2+b^2-a^2+b^2}$$

$$\frac{2x^2+2}{4x} = \frac{2a^2}{2b^2}$$

$$\Rightarrow \quad \frac{x^2+1}{2x} = \frac{a^2}{b^2}$$

$$\Rightarrow \quad b^2 = \frac{2a^2x}{x^2+1}$$

Proved

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