ICSE Paper 2009

MATHEMATICS

SECTION A [40 Marks]

*(Answer **all** questions from this Section.)*

Question 1.

(a) *Mr. Dubey borrows ₹ 1,00,000 from State Bank of India at 11% per annum compound interest. He repays ₹ 41,000 at the end of the first year and ₹ 47,700 at the end of the second year. Find the amount outstanding at the beginning of the third year.* **[3]**

(b) *A dice is thrown once. What is the probability that the*

(i) *number is even*

(ii) *number is greater than 2 ?* **[3]**

(c) *Find the HCF and LCM of the following polynomials :* **

$$3x^3 - 27x^2 + 60x \quad \text{and} \quad x^2 - 16$$ **[4]**

Solution :

(a) *Given* : P = ₹ 1,00,000, R = 11%

$$\text{Interest for first year} = \frac{PRT}{100} = \frac{1,00,000 \times 11 \times 1}{100} = ₹\ 11,000$$

$$\text{Amount after first year} = 1,00,000 + 11,000 = ₹\ 1,11,000$$

$$\text{Principal for second year} = ₹\ 1,11,000 - ₹\ 41,000$$

$$= ₹\ 70,000$$

$$\text{Interest for second year} = \frac{70,000 \times 11 \times 1}{100} = ₹\ 7,700$$

$$\text{Amount after second year} = ₹\ 70,000 + 7,700 = 77,700$$

Amount outstanding for beginning of third year

$$= ₹\ 77,700 - ₹\ 47,700$$

$$= ₹\ 30,000.$$ **Ans.**

(b) Dice is thrown once.

$$\text{Sample space} = \{1, 2, 3, 4, 5, 6\}$$

$$\therefore \quad n(S) = 6$$

(i) $$\text{Number is even} = \{2, 4, 6\}$$

$$\therefore \quad n(E) = 3$$

$$P(\text{Even number}) = \frac{n(E)}{n(S)} = \frac{3}{6} = 1/2$$ **Ans.**

(ii) $$\text{Number is greater than 2} = \{3, 4, 5, 6\}$$

$$n(E) = 4$$

$$P(> 2) = \frac{n(E)}{n(S)} = \frac{4}{6} = \frac{2}{3}$$ **Ans.**

** *Solution has not given due to out of present syllabus.*

Question 2.

(a) *Find x and y, if* $\begin{bmatrix} 2x & x \\ y & 3y \end{bmatrix}\begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 16 \\ 9 \end{bmatrix}$ **[3]**

(b) *What least number must be added to each of the numbers 5, 11, 19 and 37 so that they are in proportion ?* **[3]**

(c) *Given that $x + 2$ and $x + 3$ are factors of $2x^3 + ax^2 + 7x - b$. Determine the values of a and b.* **[4]**

Solution :

(a) *Given :*

$$\begin{bmatrix} 2x & x \\ y & 3y \end{bmatrix}\begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 16 \\ 9 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 6x + 2x \\ 3y + 6y \end{bmatrix} = \begin{bmatrix} 16 \\ 9 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 8x \\ 9y \end{bmatrix} = \begin{bmatrix} 16 \\ 9 \end{bmatrix}$$

$$\Rightarrow \left.\begin{aligned} 8x = 16 \Rightarrow x = 2 \\ 9y = 9 \Rightarrow y = 1 \end{aligned}\right]$$ **Ans.**

(b) Let the number x be added to each number.

$$5 + x : 11 + x = 19 + x : 37 + x$$

$$\frac{5 + x}{11 + x} = \frac{19 + x}{37 + x}$$

By componendo and dividendo,

$$\Rightarrow \frac{5 + x + 11 + x}{5 + x - 11 - x} = \frac{19 + x + 37 + x}{19 + x - 37 - x}$$

$$\Rightarrow \frac{16 + 2x}{-6} = \frac{56 + 2x}{-18}$$

$$\Rightarrow 3(16 + 2x) = 56 + 2x$$

$$\Rightarrow 48 + 6x = 56 + 2x$$

$$\Rightarrow 4x = 8$$

$$\Rightarrow x = 2$$ **Ans.**

(c) *Given* : $(x + 2)$ and $(x + 3)$ are the factors of $2x^3 + 9x^2 + 7x - b$.

$\because f(-2)$ and $f(-3)$ will be zero.

$$f(x) = 2x^3 + ax^2 + 7x - b$$

$$f(-2) = 2(-2)^3 + a(-2)^2 + 7(-2) - b = 0$$

$$-16 + 4a - 14 - b = 0$$

$$4a - b = 30 \quad ...(1)$$

$$f(-3) = 2(-3)^3 + a(-3)^2 + 7(-3) - b = 0$$

$$-54 + 9a - 21 - b = 0$$

$$9a - b = 75 \quad ...(2)$$

Solving (1) and (2), we get

$$a = 9,\ b = 6$$ **Ans.**

Question 3.

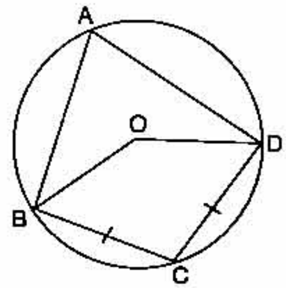
(a) *Solve the inequation and represent the solution set on the number line.*

$$-3 + x \le \frac{8x}{3} + 2 \le \frac{14}{3} + 2x, \text{ where } x \in I \qquad [3]$$

(b) *Find the value of p for which the lines*

$2x + 3y - 7 = 0$ *and* $4y - px - 12 = 0$ *are perpendicular to each other.* **[3]**

(c) *In the given figure O is the centre of the circle,* $\angle BAD = 75°$ *and chord BC = chord CD. Find : (i)* $\angle BOC$ *(ii)* $\angle OBD$ *(iii)* $\angle BCD$. **[4]**

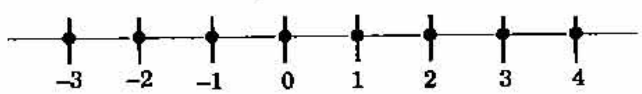


Solution :

(a)
$$-3 + x \le \frac{8x}{3} + 2 \le \frac{14}{3} + 2x, x \in I$$

$-3 + x \le \frac{8x}{3} + 2$	$\frac{8x}{3} + 2 \le \frac{14}{3} + 2x$
$\frac{8x}{3} - \frac{x}{1} \ge -3 - 2$	$\frac{8x}{3} - 2x \le \frac{14}{3} - \frac{2}{1}$
$\frac{8x - 3x}{3} \ge -5$	$\frac{8x - 6x}{3} \le \frac{14 - 6}{3}$
$5x \ge -15$	$\frac{2x}{3} \le \frac{8}{3}$
$x \ge -3$	$x \le 4$

Solution set : $\{-3 \le x \le 4\}$



(b) Given equation is $2x + 3y - 7 = 0$

$\Rightarrow$ $3y = -2x + 7$

$\Rightarrow$ $y = -\frac{2}{3}x + \frac{7}{3}$

Slope of the line $(m_1) = -\frac{2}{3}$

Another equation is $4y - px - 12 = 0$

$\Rightarrow$ $4y = px + 12$

$\Rightarrow$ $y = \frac{p}{4}x + 3$

Slope of the line $(m_2) = \frac{p}{4}$

As per the question, lines are perpendicular.

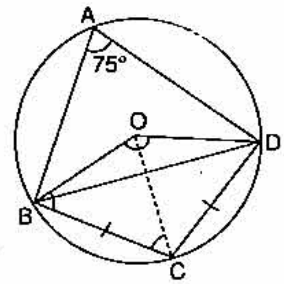
$$m_1 \times m_2 = -1$$

$$-\frac{2}{3} \times \frac{p}{4} = -1$$

$$\Rightarrow \quad -2p = -12$$

$$\Rightarrow \quad p = 6$$ **Ans.**

(c) *Given* : $\angle BAD = 75°$, chord BC = chord CD



$$\angle BOD = 2 \times \angle BAD = 2 \times 75° = 150°$$

(i) $$\angle BOC = \frac{1}{2} \angle BOD$$

$$= \frac{1}{2} \times 150° = 75°$$ **Ans.**

(ii) $$\angle OBD = \frac{1}{2}(180° - 150°)$$

$$= 15°$$ **Ans.**

(iii) $$\angle BCD = 180° - 75° = 105°$$ **Ans.**

Question 4.

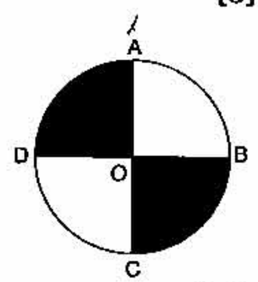
(a) *Find the mean, median and mode of the following distribution :*

8, 10, 7, 6, 10, 11, 6, 13, 10 **[3]**

(b) *Without using trigonometric tables evaluate the following :*

$$\frac{\sec 17°}{\text{cosec } 73°} + \frac{\tan 68°}{\cot 22°} + \cos^2 44° + \cos^2 46°$$ **[3]**

(c) *AC and BD are two perpendicular diameters of a circle with centre O. If AC = 16 cm, calculate the area and perimeter of the shaded part. (Take $\pi = 3.14$)* **[4]**



Solution :

(a) $$\text{Mean} = \frac{\Sigma x}{n} = \frac{8 + 10 + 7 + 6 + 10 + 11 + 6 + 13 + 10}{9}$$

$$= \frac{81}{9} = 9$$ **Ans.**

For the median, we arrange the data in ascending order

6, 6, 7, 8, 10, 10, 10, 11, 13

$$\text{Median} = \left(\frac{n+1}{2}\right)^{th} \text{term} = \left(\frac{9+1}{2}\right)^{th} \text{term}$$

$$= 5^{th} \text{ term} = 10$$

In the given data, 10 occurs maximum number of times, therefore

$$\text{Mode} = 10$$ **Ans.**

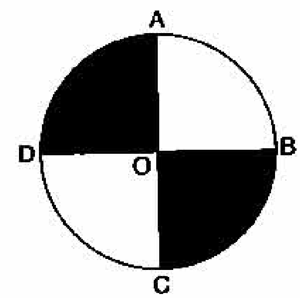
(b) *Given* : $$\frac{\sec 17°}{\text{cosec } 73°} + \frac{\tan 68°}{\cot 22°} + \cos^2 44° + \cos^2 46°$$

$$= \frac{\sec(90° - 73°)}{\operatorname{cosec} 73°} + \frac{\tan(90° - 22°)}{\cot 22°} + \cos^2(90° - 46°) + \cos^2 46°$$

$$= \frac{\operatorname{cosec} 73°}{\operatorname{cosec} 73°} + \frac{\cot 22°}{\cot 22°} + \sin^2 46° + \cos^2 46°$$

$$= 1 + 1 + 1 = 3$$ **Ans.**

(c) *Given* : AC = 16 cm = diameter of the circle, $\pi = 3{\cdot}14$.



$$\text{Area of shaded portion} = 2 \text{ quadrant}$$

$$= \frac{1}{2} \times \pi r^2$$

$$= \frac{1}{2} \times 3{\cdot}14 \times (8)^2$$

$$= 100{\cdot}48 \text{ cm}^2$$ **Ans.**

$$\text{Perimeter of the shaded portion} = \pi r + DB + AC$$

$$= 3{\cdot}14 \times 8 + 16 + 16$$

$$= 57{\cdot}12 \text{ cm.}$$ **Ans.**

SECTION B [40 Marks]

*Answer any **four** Questions in this Section.*

Question 5.

(a) *A shopkeeper bought a TV at a discount of 30% of the listed price of ₹ 24,000. The shopkeeper offers a discount of 10% of the listed price to his customer. If the VAT (Value Added Tax) is 10%.*

Find : (i) *the amount paid by the customer.*

(ii) *the VAT to be paid by the shopkeeper.* **[3]**

(b) *Solve the following quadratic equation and give the answer correct to two significant figures.*

$$4x^2 - 7x + 2 = 0$$ **[3]**

(c) *Use graph paper to answer this question.*

(i) *Plot the points A (4, 6) and B (1, 2)*

(ii) *A′ is the image of A when reflected in X-axis.*

(iii) *B′ is the image of B when B is reflected in the line AA′.*

(iv) *Give the geometrical name for the figure ABA′B′.* **[4]**

Solution :

(a) (i)

$$\text{Discount} = 30\% \text{ on } 24{,}000$$

$$= \frac{30}{100} \times 24{,}000 = ₹\, 7{,}200$$

$$\text{Cost price of shopkeeper} = 24{,}000 - 7{,}200$$

$$= ₹\, 16{,}800$$

$$\text{Tax @ } 10\% = \frac{10 \times 16800}{100} = ₹\, 1680$$

$$\text{Amount paid by shopkeeper} = 16{,}800 + 1{,}680 = ₹\, 18{,}480$$

$$\text{Discount on customer} = 10\% \text{ on } 24{,}000 = \frac{10}{100} \times 24{,}000$$
$$= ₹\,2{,}400$$
$$\text{Selling price of shopkeeper} = 24{,}000 - 2{,}400 = ₹\,21{,}600$$
$$\text{Tax @ } 10\% = \frac{10 \times 21{,}600}{100} = ₹\,2{,}160$$
$$\text{The amount paid by customer} = 21{,}600 + 2{,}160 = ₹\,23{,}760$$

Ans.

(ii)
$$\text{VAT to be paid by shopkeeper} = ₹\,2{,}160 - ₹\,1{,}680$$
$$= ₹\,480$$

Ans.

(b) Given equation $4x^2 - 7x + 2 = 0$ comparing with $ax^2 + bx + c = 0$, we have $a = 4$, $b = -7$, $c = 2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{+7 \pm \sqrt{(7)^2 - 4 \times 4 \times 2}}{2 \times 4} = \frac{+7 \pm \sqrt{49 - 32}}{8}$$
$$= \frac{7 \pm 4{\cdot}123}{8}$$

Taking $+ve$ sign

$$x = \frac{7 + 4{\cdot}123}{8} = \frac{11{\cdot}123}{8} = 13{\cdot}9$$

Taking $-ve$ sign

$$x = \frac{7 - 4{\cdot}123}{8} = \frac{2{\cdot}877}{8} = 0{\cdot}36$$

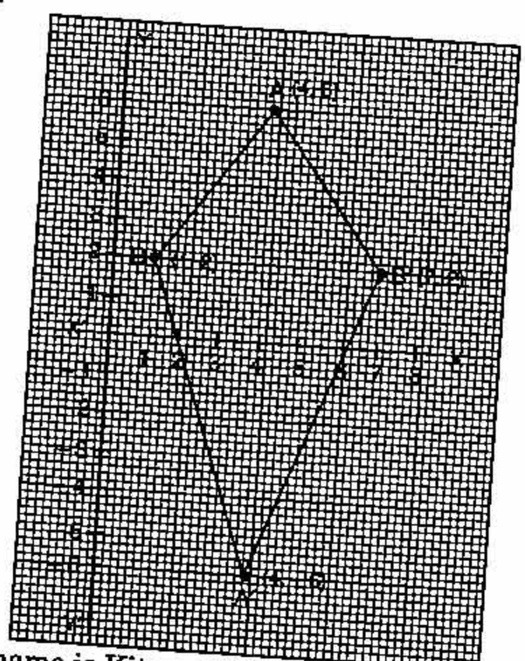
$\Rightarrow$
$$= 1{\cdot}390 \text{ and } 0{\cdot}3596$$
$$= 1{\cdot}4 \text{ and } 0{\cdot}36$$

Ans.

(c) (i) See figure.

(ii) See figure.

(iii) See figure.



(iv) Geometrical name is Kite.

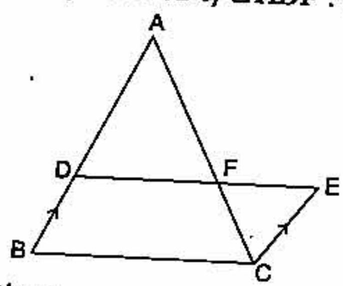
Question 6.

(a) *In the given figure, ABC and CEF are two triangles where BA is parallel to CE and AF : AC = 5 : 8.*

(i) *Prove that $\Delta ADF \sim \Delta CEF$.*

(ii) *Find AD if CE = 6 cm.*

(iii) *If DF is parallel to BC find area of ΔADF : area of ΔABC.* **[3]**



(b) *Prove the following identity :*

$$\frac{\sin A}{1+\cos A} + \frac{1+\cos A}{\sin A} = 2 \operatorname{cosec} A$$

[3]

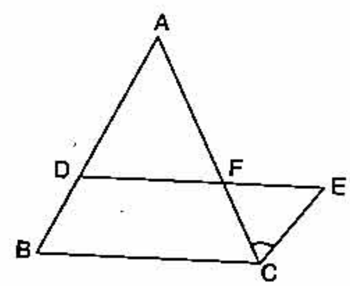
(c) *The following table gives the wages of workers in a factory :*

Wages in ₹	*45–50*	*50–55*	*55–60*	*60–65*	*65–70*	*70–75*	*75–80*
No. of workers	*5*	*8*	*30*	*25*	*14*	*12*	*6*

Calculate the mean by the short cut method. **[4]**

Solution :

(a) In Δ ABC and Δ CEF,



$$BA \parallel CE \text{ (given)}$$

and $$\frac{AF}{AC} = \frac{5}{8} \text{ (given)}$$

$$\Rightarrow \quad \frac{AF}{AF+FC} = \frac{5}{5+3}$$

$$\Rightarrow \quad \frac{AF}{FC} = \frac{5}{3}$$

(i)

$$\angle DAF = \angle FCE \quad \text{(Int. }\angle\text{)}$$

$$\angle AFD = \angle CFE \quad \text{(Vert.)}$$

$$\therefore \quad \Delta ADF \sim \Delta CEF \quad \text{(AA similarity)}$$

Proved

(ii)

$$\frac{AD}{CE} = \frac{AF}{FC} \Rightarrow \frac{AD}{6} = \frac{5}{3}$$

$$\Rightarrow \quad AD = \frac{5}{3} \times 6 = 10 \text{ cm}$$

Ans.

(iii) *Given* : DF $\parallel$ BC

$$\Delta ADF \sim \Delta ABC$$

$$\frac{AF}{AC} = \frac{AD}{AB} = \frac{5}{8}$$

$$\frac{\text{Area of } \Delta ADF}{\text{Area of } \Delta ABC} = \frac{AD^2}{AB^2} = \frac{(5)^2}{(8)^2} = \frac{25}{64}$$

Ans.

(b)

$$\text{L.H.S.} = \frac{\sin A}{1+\cos A} + \frac{1+\cos A}{\sin A}$$
$$= \frac{\sin^2 A + 1 + \cos^2 A + 2\cos A}{\sin A\,(1+\cos A)}$$
$$= \frac{2 + 2\cos A}{\sin A\,(1+\cos A)} = \frac{2\,(1+\cos A)}{\sin A(1+\cos A)}$$
$$= 2\,\text{cosec}\,A = \text{R.H.S.}$$

Proved

(c)

Class interval	*Class Marks* x_i	$d_i = x_i - A$	*Frequency* f_i	$f_i.d_i$
45–50	47·5	– 15	5	– 75
50–55	52·5	–10	8	– 80
55–60	57·5	–5	30	– 150
60–65	62·5 (A)	0	25	0
65–70	67·5	5	14	70
70–75	72·5	10	12	120
75–80	77·5	15	6	90
			$\Sigma f_i = 100$	$\Sigma f_i d_i = -25$

Here, A = 62·5

$$\text{Mean} = A + \frac{\Sigma f_i d_i}{\Sigma f_i} = 62{\cdot}5 + \left(\frac{-25}{100}\right)$$
$$= 62{\cdot}5 - 0{\cdot}25 = 62{\cdot}25$$

Ans.

Question 7.

(a) *Amit Kumar invests ₹ 36,000 in buying ₹ 100 shares at ₹ 20 premium. The dividend is 15% per annum. Find :*

(i) *The number of shares he buys*

(ii) *His yearly dividend*

(iii) *The percentage return on his investment.*

Give your answer correct to the nearest whole number. **[3]**

(b) *What sum of money will amount to ₹ 9,261 in 3 years at 5% per annum compound interest ?* **[3]**

(c) *Mr. Mishra has a Savings Bank Account in Allahabad Bank. His pass book entries are as follows :*

Date	*Particulars*	*Withdrawals (in ₹)*	*Deposits (in ₹)*	*Balance (in ₹)*
Jan. 4, 2007	*By Cash*	—	*1000.00*	*1000.00*
Jan. 11, 2007	*By Cheque*	—	*3000.00*	*4000.00*
Feb. 3, 2007	*By Cash*	—	*2500.00*	*6500.00*
Feb. 7, 2007	*To Cheque*	*2000.00*	—	*4500.00*
March 3, 2007	*By Cash*	—	*5000.00*	*9500.00*
March 25, 2007	*By Cash*	—	*2000.00*	*11,500.00*
June 7, 2007	*By Cash*	—	*3500.00*	*15000.00*
Aug. 29, 2007	*To Cheque*	*1000.00*	—	*14000.00*

Rate of interest paid by the bank is 4.5% per annum. Mr. Mishra closes his account on 30th October, 2007. Find the interest he receives. **[4]**

Solution :

(a)

$$\text{MV of 1 share} = ₹\,(100 + 20) = ₹\,120$$

Given : Dividend = 15%, Investment = 36,000

(i)
$$\text{Number of shares buys} = \frac{\text{Investment}}{\text{MV}}$$
$$= \frac{36{,}000}{120} = 300$$ **Ans.**

(ii)
$$\text{Dividend on 1 share} = ₹\,\frac{15}{100} \times 100 = ₹\,15$$
$$\text{Dividend on 300 shares} = 15 \times 300$$
$$= ₹\,4{,}500$$ **Ans.**

(iii)
$$\text{Rate of interest} = \frac{4{,}500}{36{,}000} \times 100$$
$$= 12{\cdot}5\%$$ **Ans.**

(b) *Given* A = ₹ 9,261, T = 3 year, R = 5%

$$A = P\left(1 + \frac{R}{100}\right)^T$$

$$\Rightarrow \quad 9{,}261 = P\left(1 + \frac{5}{100}\right)^3$$

$$\Rightarrow \quad 9{,}261 = P\left(\frac{21}{20}\right)^3$$

$$\Rightarrow \quad P = \frac{9{,}261 \times 20 \times 20 \times 20}{21 \times 21 \times 21}$$

$$= ₹\,8{,}000$$ **Ans.**

(c) Qualifying amounts for interest for various months :

January	₹	1,000
February	₹	4,500
March	₹	9,500
April	₹	9,500
May	₹	9,500
June	₹	15,000
July	₹	15,000
August	₹	14,000
September	₹	14,000
	₹	92,000

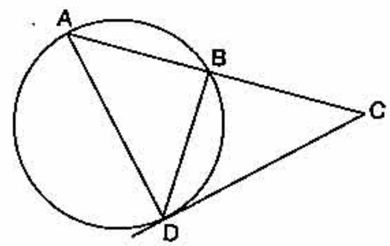
$$P = ₹\,92{,}000$$
$$T = 1 \text{ month}$$
$$R = 4{\cdot}5\%$$
$$\text{Interest} = \frac{P \times R \times T}{100}$$
$$= \frac{92{,}000 \times 4{\cdot}5 \times 1}{100 \times 12}$$
$$= ₹\,345$$ **Ans.**

Question 8.

(a) *Given that* $\frac{a^3+3ab^2}{b^3+3a^2b}=\frac{63}{62}$.

Using Componendo and Dividendo find a : b. **[3]**

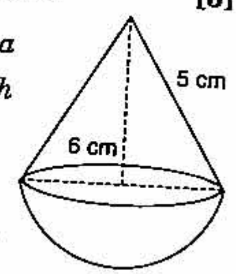
(b)



In the above figure AB = 7 cm and BC = 9 cm.

(i) *Prove* $\Delta ACD \sim \Delta DCB$. (ii) *Find the length of CD.* **[3]**

(c) *The given figure represents a hemisphere surmounted by a conical block of wood. The diameter of their bases is 6 cm each and the slant height of the cone is 5 cm. Calculate :*



(i) *the height of the cone.*

(ii) *the volume of the solid.* **[4]**

Solution :

(a) *Given :*

$$\frac{a^3+3ab^2}{b^3+3a^2b}=\frac{63}{62}$$

By componendo and dividendo,

$$\frac{a^3+3ab^2+b^3+3a^2b}{a^3+3ab^2-b^3-3a^2b}=\frac{63+62}{63-62}$$

$$\Rightarrow \quad \frac{(a+b)^3}{(a-b)^3}=\frac{125}{1}=\left(\frac{5}{1}\right)^3$$

$$\Rightarrow \quad \frac{a+b}{a-b}=\frac{5}{1}$$

Again componendo and dividendo,

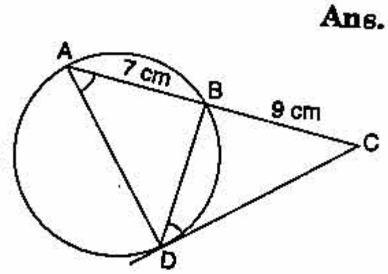
$$\Rightarrow \quad \frac{a+b+a-b}{a+b-a+b}=\frac{5+1}{5-1}=\frac{6}{4}$$

$$\Rightarrow \quad \frac{2a}{2b}=\frac{3}{2}$$

$$\Rightarrow \quad a:b=3:2$$

Ans.

(b) *Given* : AB = 7 cm, BC = 9 cm.



(i) In Δ ACD and Δ DCB,

$\angle C = \angle C$ (common)

$\angle CDB = \angle BAD$ ($\angle$'s alternate segment)

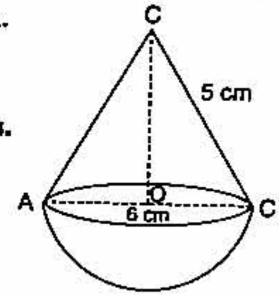
$\Delta ACD \sim \Delta DCB$ (By A·A similarity)

Proved

(ii)
$$AC \times BC = CD^2 \quad (\because \Delta ACD \sim \Delta DCB)$$
$$CD^2 = 16 \times 9 = 144$$
$$CD = 12 \text{ cm.}$$
Ans.

(c) *Given* : Diameter (AB) = 6 cm, r (OB) = 3 cm, l (DB) = 5 cm.

(i)
$$OD^2 = DB^2 - OB^2$$
$$= 25 - 9 = 16$$
$$OD = 4 \text{ cm}$$
Ans.



(ii)
$$\text{Volume of hemisphere} = \frac{2}{3}\pi r^3$$
$$= \frac{2}{3} \times \frac{22}{7} \times (3)^3$$
$$= 56{\cdot}57 \text{ cm}^3.$$
$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 9 \times 4 = 37{\cdot}71 \text{ cm}^3$$
$$\text{Volume of the solid} = 37{\cdot}71 + 56{\cdot}57$$
$$= 94{\cdot}28 \text{ cm}^3.$$
Ans.

Question 9.

(a) *Attempt this question on graph paper.*

Marks obtained by 200 students in examination are given below :

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
No. of Students	5	10	14	21	25	34	36	27	16	12

Draw an Ogive for the given distribution taking 2 cm = 10 marks on one axis and 2 cm = 20 students on the other axis.

From the graph find :

(i) *the Median*

(ii) *the Upper Quartile*

(iii) *Number of students scoring above 65 marks.*

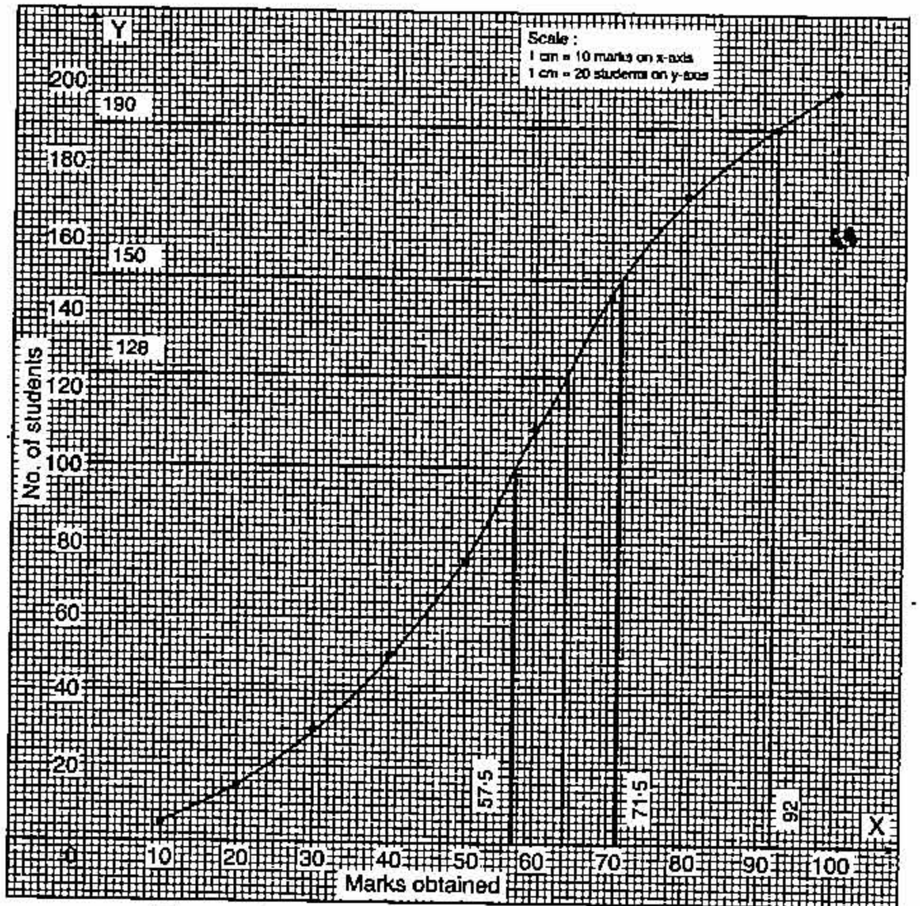
(iv) *If 10 students qualify for merit scholarship, find the minimum marks required to qualify.* **[6]**

(b) *From two points A and B on the same side of a building, the angles of elevation of the top of the building are 30° and 60° respectively. If the height of the building is 10m, find the distance between A and B correct to two decimal places.* **[4]**

Solution :

(a)

Marks	No. of students	*cf*
0–10	5	5
10–20	10	15
20–30	14	29
30–40	21	50
40–50	25	75
50–60	34	109
60–70	36	145
70–80	27	172
80–90	16	188
90–100	12	200



(i) Here n (no. of students) $= 200$ (even)

$\therefore$ Median $= \left(\frac{n}{2}\right)^{th}$ term $= 100$th term.

From the graph 100th term is 57·5.

Median $= 57{\cdot}5$ **Ans.**

(ii) Upper quartile $(Q_3) = \frac{3n}{4} = \frac{3 \times 200}{4}$th term

$= 3 \times 50 = 150$ term

From Graph 150th term $= 71{\cdot}5$

The upper quartile $= 71{\cdot}5$ **Ans.**

(iii) Number of students scoring above 65 marks $= 200 - 128 = 72$ **Ans.**

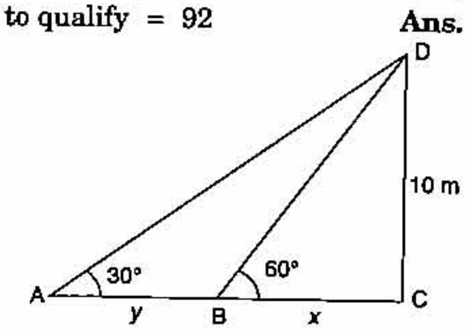
(iv) Minimum marks to qualify $= 92$ **Ans.**

(b) In Δ DBC, $\tan 60° = \frac{10}{x}$

$\Rightarrow \sqrt{3} = \frac{10}{x}$

$\Rightarrow x = \frac{10}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{10\sqrt{3}}{3}$ m



In Δ DAC, $\tan 30° = \dfrac{10}{x+y} = \dfrac{1}{\sqrt{3}}$

$$\Rightarrow \quad x + y = 10\sqrt{3}$$

$$\Rightarrow \quad y = 10\sqrt{3} - \frac{10}{\sqrt{3}}$$

$$= \frac{30-10}{\sqrt{3}} = \frac{20}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{20}{3}\sqrt{3}$$

$$= 11 \cdot 55 \text{ m}$$

Ans.

Question 10.

(a) *Mrs. Goswami deposits ₹ 1000 every month in a recurring deposit account for 3 years at 8% interest per annum. Find the matured value.* **[3]**

(b) *Find the equation of a line with x intercept = 5 and passing through the point (4, –7).* **[3]**

(c) *In a school the weekly pocket money of 50 students is as follows :*

Weekly pocket money in ₹	*40-50*	*50-60*	*60-70*	*70-80*	*80-90*	*90-100*
No. of students	*2*	*8*	*12*	*14*	*8*	*6*

Draw a histogram and a frequency polygon on the same graph. Find the mode from the graph. **[4]**

Solution :

(a) Total Principal (P) for 1 month

$$P = x \times \frac{n(n+1)}{2} = 1000 \times \frac{36 \times 37}{2}$$

$$= ₹\ 6{,}66{,}000$$

$$\text{Interest for 1 month} = \frac{PRT}{100} = \frac{6{,}66{,}000 \times 8 \times 1}{100 \times 12}$$

$$= ₹\ 4{,}440$$

$$\text{Total amount deposited by Mr. Goswami} = 36 \times 1000 = ₹\ 36{,}000$$

$$\text{Maturity value} = ₹\ 36{,}000 + 4{,}440$$

$$= ₹\ 40{,}440$$

Ans.

(b) Equation of the line passing through (5, 0) and (4, –7) :

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 0 = \frac{-7-0}{4-5}(x-5)$$

$$y = \frac{-7}{-1}(x-5)$$

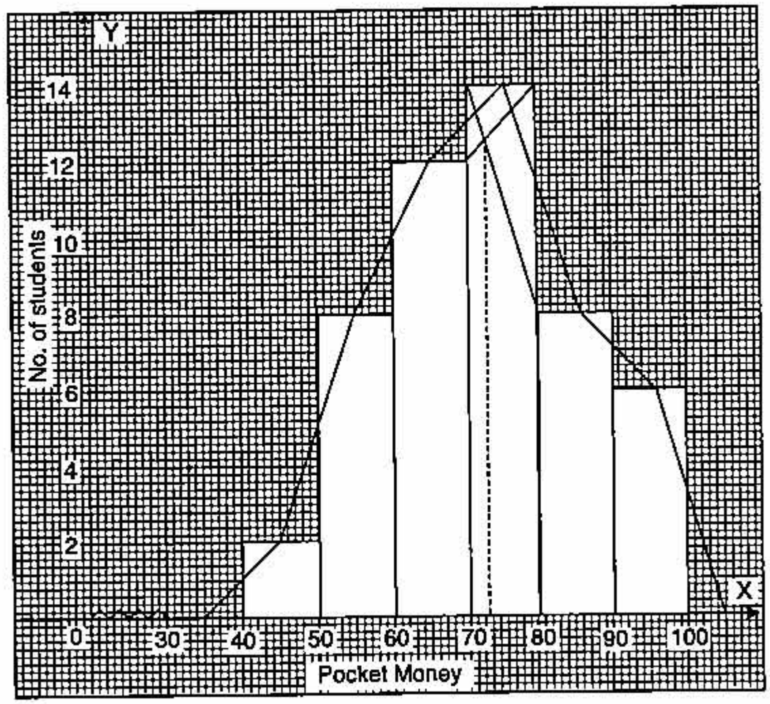
$$\Rightarrow \quad y = 7(x-5)$$

$$\Rightarrow \quad y = 7x - 35$$

$$\Rightarrow \quad 7x - y - 35 = 0$$

Ans.

(c)



So, from the graph, Mode = 73 **Ans.**

Question 11.

(a) *The model of a building is constructed with scale factor 1 : 30.*

(i) *If the height of the model is 80 cm, find the actual height of the building in metres.*

(ii) *If the actual volume of a tank at the top of the building is $27 m^3$, find the volume of the tank on the top of the model.* **[2]**

(b) *The speed of an express train is x km/h and the speed of an ordinary train is 12 km/h less than that of the express train. If the ordinary train takes one hour longer than the express train to cover a distance of 240 km, find the speed of the express train.* **[4]**

(c) *Using ruler and compasses construct*

(i) *a triangle ABC in which AB = 5.5 cm, BC = 3.4 cm and CA = 4.9 cm.*

(ii) *the locus of points equidistant from A and C.*

(iii) *a circle touching AB at A and passing through C.* **[4]**

Solution :

(a) Scale factor $k = \frac{1}{30}$

(i) Height of the model = k (times the height of the building)

$\Rightarrow$ Height of building = 80×30

= 2400 cm

= 24 m **Ans.**

(ii) Volume of model $= k^3$ times volume of the building

$$= \frac{1}{30}\times\frac{1}{30}\times\frac{1}{30}\times 27\text{ m}^3$$

$$= \frac{1}{1000}\text{ m}^3 = 0{\cdot}001\text{ m}^3$$ **Ans.**

(b) Time taken by express train $= \dfrac{240}{x}$ h

Speed of ordinary train $= (x-12)$ km/h

Time taken by ordinary train $= \dfrac{240}{x-12}$

According to the question, $\dfrac{240}{x} = \dfrac{240}{x-12} - 1$

$$\Rightarrow \quad 240(x-12) = 240x - x(x-12)$$

$$\Rightarrow \quad 240x - 2880 = 240x - x^2 + 12x$$

$$\Rightarrow \quad x^2 - 12x - 2880 = 0$$

$$\Rightarrow \quad x^2 - 60x + 48x - 2880 = 0$$

$$\Rightarrow \quad x(x-60) + 48(x-60) = 0$$

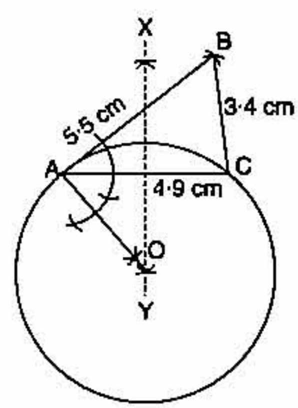
$$\Rightarrow \quad (x-60)(x+48) = 0$$

either $x - 60 = 0$ or $x + 48 = 0$

$\Rightarrow$ $x = 60$ or $x = -48$ **Ans.**

Hence, the speed of express train is 60 km/h.

(c) Steps of construction :



1. Draw Δ ABC with given values.
2. Draw XY perpendicular bisector of AC.
3. Draw perpendicular of AB at A which cuts perpendicular XY at O.
4. Draw a circle at centre O which touching AB at A and passing through C *i.e.*, required circle.