

# ICSE Paper 2006

## MATHEMATICS

### SECTION A [40 Marks]

(Answer all questions from this Section.)

#### Question 1.

- (a) Kiran purchases an article for ₹ 5,400 which includes 10% rebate on the marked price and 20% sales tax on the remaining price. Find the marked price of the article. [3]
- (b) If  $\frac{3x + 5y}{3x - 5y} = \frac{7}{3}$ , find  $x : y$ . [3]
- (c) A person invests ₹ 10,000 for two years at a certain rate of interest compounded annually. At the end of one year this sum amounts to ₹ 12,000. Calculate : [4]
- (i) the rate of interest per annum.
  - (ii) the amount at the end of the second year.

#### Solution.

- (a) Let the M.P. of the article be ₹  $x$

$$\therefore \text{Rebate} = 10\% \text{ of M.P.} = \frac{10}{100}x$$

$$\text{Selling price} = \text{MP} - \text{Rebate}$$

$$= x - \frac{10x}{100} = ₹ \frac{90x}{100}$$

$$\therefore \text{Sales tax} = 20\% \text{ of selling price}$$

$$= \frac{20}{100} \times \frac{90x}{100}$$

$$= ₹ \frac{18x}{100}$$

$$\text{Final price} = \text{SP} + \text{Sales tax}$$

$$5400 = \frac{90x}{100} + \frac{18x}{100}$$

$$5400 = \frac{108x}{100}$$

$$x = \frac{5400 \times 100}{108}$$

$$x = ₹ 5000.$$

Ans.

(b) Given :  $\frac{3x + 5y}{3x - 5y} = \frac{7}{3}$

Apply componendo and dividendo

$$\frac{3x + 5y + 3x - 5y}{3x + 5y - 3x + 5y} = \frac{7 + 3}{7 - 3}$$

$$\frac{6x}{10y} = \frac{10}{4}$$

$$\frac{x}{y} = \frac{10 \times 10}{4 \times 6} = \frac{25}{6}$$

$$\therefore x : y = 25 : 6$$

Ans.

(c) Given :  $P_1 = ₹ 10,000$ ,  $T = 2$  yrs.  $R = ?$

Amount after 1st year = ₹ 12,000

∴ Interest for 1st year =  $12,000 - 10,000 = ₹ 2,000$

$$(i) \quad I_1 = \frac{P_1 RT}{100}$$

$$\therefore R = \frac{I_1 \times 100}{P_1 \times T} = \frac{2,000 \times 100}{10,000 \times 1}$$

$$= 20\% \quad \text{Ans.}$$

(ii)  $P_2$  for 2nd year = ₹ 12,000

$$\therefore I_2 \text{ for 2nd year} = \frac{P_2 RT}{100} = \frac{12,000 \times 20 \times 1}{100}$$

$$= ₹ 2,400$$

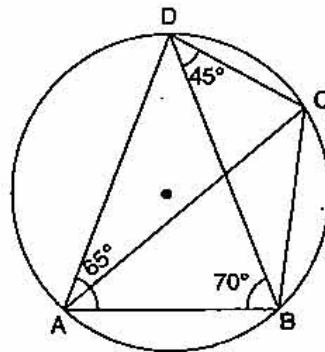
$$\begin{aligned} \text{Amount at the end of 2nd year} &= P_2 + I_2 \\ &= 12,000 + 2,400 \\ &= ₹ 14,400. \end{aligned} \quad \text{Ans.}$$

### Question 2.

(a) Show that  $2x + 7$  is a factor of  $2x^3 + 5x^2 - 11x - 14$ . Hence factorise the given expression completely, using the factor theorem. [3]

(b) The median of the following observation 11, 12, 14, 18,  $(x + 4)$ , 30, 32, 35, 41 arranged in ascending order is 24. Find  $x$ . [3]

(c) In the given figure  $\angle BAD = 65^\circ$ ,  $\angle ABD = 70^\circ$ , and  $\angle BDC = 45^\circ$ . Find : (i)  $\angle BCD$ , (ii)  $\angle ADB$ .



Hence show that  $AC$  is a diameter. [4]

### Solution.

(a) If  $2x + 7$  is a factor of  $2x^3 + 5x^2 - 11x - 14$  then

$$2x + 7 = 0$$

$$x = \frac{-7}{2}$$

Let

$$f(x) = 2x^3 + 5x^2 - 11x - 14$$

if  $2x + 7$  is a factor of  $f(x)$  then

$$f\left(\frac{-7}{2}\right) = 0$$

$$= 2\left(\frac{-7}{2}\right)^3 + 5\left(\frac{-7}{2}\right)^2 - 11\left(\frac{-7}{2}\right) - 14$$

$$\begin{aligned}
 &= \frac{-343}{4} + \frac{245}{4} + \frac{77}{2} - 14 \\
 &= \frac{-343 + 245 + 154 - 56}{4} \\
 &= \frac{-399 + 399}{4} = \frac{0}{4} = 0
 \end{aligned}$$

Hence  $2x + 7$  is a factor of  $f(x)$ .

Now by applying factor theorem

$$\begin{array}{r}
 x^2 - x - 2 \\
 2x + 7 \overline{) 2x^3 + 5x^2 - 11x - 14} \\
 \underline{2x^2 + 7x^2} \phantom{- 11x - 14} \\
 -2x^2 - 11x \phantom{- 14} \\
 \underline{-2x^2 - 7x} \phantom{- 14} \\
 + \phantom{+} \phantom{- 14} \\
 -4x - 14 \\
 \underline{-4x - 14} \\
 + \phantom{+} \\
 0
 \end{array}$$

$\therefore$  factors of

$$\begin{aligned}
 &2x^3 + 5x^2 - 11x - 14 \\
 &= (2x + 7)(x^2 - x - 2) \\
 &= (2x + 7)(x^2 - 2x + x - 2) \\
 &= (2x + 7)[x(x - 2) + 1(x - 2)] \\
 &= (2x + 7)(x + 1)(x - 2)
 \end{aligned}$$

Ans.

(b) Given : 11, 12, 14, 18m,  $(x + 4)$ , 30, 32, 35, 41

No. of observations = 9

$\therefore n$  is odd.

$$\therefore \text{Median} = \frac{n+1}{2} \text{th term}$$

$$= \frac{9+1}{2} \text{th term} = 5\text{th term}$$

$$\therefore x + 4 = 24$$

$$x = 20.$$

Ans.

(c) (i)  $\therefore$  ABCD is a cyclic quadrilateral.

$$\therefore \angle DAB + \angle BCD = 180^\circ$$

[ $\therefore$  sum of opp. angles in cyclic quadrilateral is  $180^\circ$ ]

$$65^\circ + \angle BCD = 180^\circ$$

$$\therefore \angle BCD = 180^\circ - 65^\circ$$

$$= 115^\circ.$$

Ans.

(ii) In  $\triangle ABD$ ,

$$\angle DAB = 65^\circ$$

$$\angle DBA = 70^\circ$$

$$\angle ADB = 180^\circ - (\angle DAB + \angle DBA) \quad [\text{Sum of angles of a } \triangle]$$

$$\angle ADB = 180^\circ - (70^\circ + 65^\circ)$$

$$= 180^\circ - 135^\circ$$

$$\therefore \angle ADB = 45^\circ.$$

**Ans.**

Now,

$$\angle ADC = \angle ADB + \angle BDC$$

$$= 45^\circ + 45^\circ$$

$$= 90^\circ \quad \therefore \text{angle in a semi circle is equal to } 90^\circ.$$

$\therefore$  AC is diameter.

**Hence Proved.**

### Question 3.

(a) Mohan deposits ₹ 80 per month in a cumulative deposit account for six years. Find the amount payable to him on maturity, if the rate of interest is 6% per annum. [3]

(b) A rectangular playground has two semicircles added to its outside its smaller sides as diameters. If the sides of the rectangle are 120 m and 21 m, find the area of the playground. ( $\pi = 22/7$ ). [3]

(c) Use graph paper for this question.

The points A (2, 3), B (4, 5) and C (7, 2) are the vertices of  $\triangle ABC$ .

(i) Write down the coordinates of A', B', C' if  $\triangle A'B'C'$  is the image of  $\triangle ABC$ , when reflected in the origin.

(ii) Write down the co-ordinates of A'', B'', C'' if  $A''B''C''$  is the image of  $\triangle ABC$ , when reflected in the x-axis.

(iii) Mention the special name of the quadrilateral  $BBC''B''$  and find its area.

[4]

### Solution.

(a) Given : P per month = ₹ 80, Time ( $n$ ) = 6 years =  $6 \times 12 = 72$  months, R = 6%

$$\text{Equivalent principal for 72 months} = 80 \times \frac{n(n+1)}{2}$$

$$= 80 \times \frac{72 \times 73}{2}$$

$$= ₹ 2,10,240$$

$$\text{Interest} = \frac{PRT}{100} = \frac{210240 \times 6 \times 1}{100 \times 12}$$

$$= ₹ 1051.20$$

$$\text{Maturity Amount} = Pn + \text{Interest}$$

$$= 80 \times 72 + 1051.20$$

$$= 5760 + 1051.20$$

$$= ₹ 6811.20$$

**Ans.**

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(b) Area of playground = Area of rectangle + Area of two semicircles

$$= l \times b + 2 \left( \frac{\pi r^2}{2} \right)$$

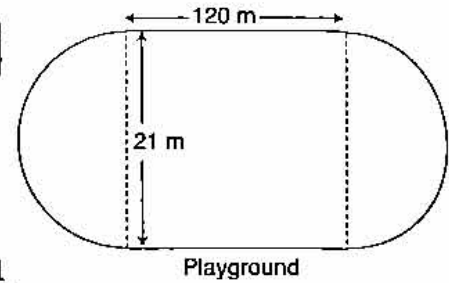
Given :  $l = 120 \text{ m}$ ,  $b = 21 \text{ m}$ ,  $r = \frac{21}{2} \text{ m}$

$\therefore$  Area of playground =  $120 \times 21 +$

$$\frac{2 \times 22 \times 21 \times 21}{2 \times 7 \times 2 \times 2}$$

$$= 2520 + 346.50$$

$$= 2866.5 \text{ m}^2$$



Ans.

(c) Given : A (2, 3), B (4, 5), C (7, 2)

(i) Reflection in origin

$$(x, y) \xrightarrow{M_o} (-x, -y)$$

$$\therefore A (2, 3) \xrightarrow{M_o} A' (-2, -3)$$

$$B (4, 5) \xrightarrow{M_o} B' (-4, -5)$$

$$C (7, 2) \xrightarrow{M_o} C' (-7, -2)$$

(ii) Reflection in x axis

$$(x, y) \xrightarrow{M_x} (x, -y)$$

$$A (2, 3) \xrightarrow{M_x} A'' (2, -3)$$

$$B (4, 5) \xrightarrow{M_x} B'' (4, -5)$$

$$C (7, 2) \xrightarrow{M_x} C'' (7, -2)$$

(iii) BCC''B'' is an isosceles trapezium.

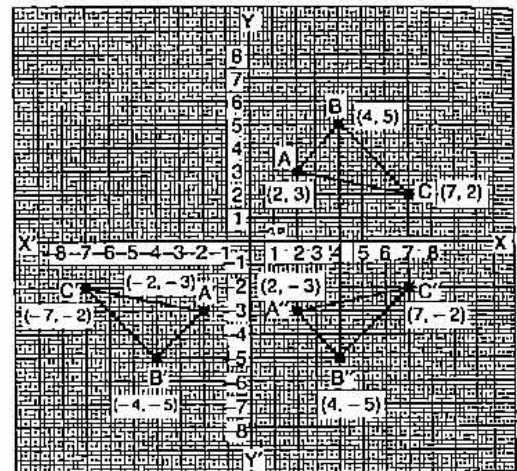
$$\text{Area of trapezium} = \frac{1}{2} (b_1 + b_2) \times h$$

Here  $b_1 = 4$  units,  $b_2 = 10$  units,  $h = 3$  units.

$$= \frac{1}{2} (4 + 10) \times 3$$

$$= 21 \text{ unit}^2.$$

Ans.



**Question 4.**

(a) Without using tables, evaluate :  $\frac{2 \tan 53^\circ}{\cot 37^\circ} - \frac{\cot 80^\circ}{\tan 10^\circ}$  [3]

(b) Given that  $x \in R$ , solve the following inequality and graph the solution on the number line :  $-1 \leq 3 + 4x < 23$  [3]

(c) Find the mean of the following distribution : [4]

|                |       |       |       |       |       |       |
|----------------|-------|-------|-------|-------|-------|-------|
| Class interval | 20—30 | 30—40 | 40—50 | 50—60 | 60—70 | 70—80 |
| Frequency      | 10    | 6     | 8     | 12    | 5     | 9     |

**Solution.**

(a) Given :

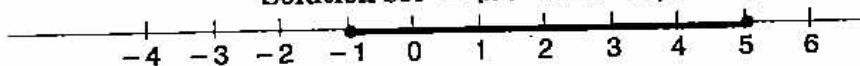
$$\begin{aligned} & \frac{2 \tan 53^\circ}{\cot 37^\circ} - \frac{\cot 80^\circ}{\tan 10^\circ} \\ &= \frac{2 \tan 53^\circ}{\cot (90^\circ - 53^\circ)} - \frac{\cot 80^\circ}{\tan (90^\circ - 80^\circ)} \\ &= \frac{2 \tan 53^\circ}{\tan 53^\circ} - \frac{\cot 80^\circ}{\cot 80^\circ} \\ &= 2 - 1 \\ &= 1 \end{aligned}$$

**Ans.**

(b) Given :

$$\begin{aligned} & -1 \leq 3 + 4x < 23 \\ & -1 \leq 3 + 4x \quad \text{or} \quad 3 + 4x < 23 \\ & -4 \leq 4x \quad \text{or} \quad 4x < 20 \\ & -1 \leq x \quad \text{or} \quad x < 5 \end{aligned}$$

$$\text{Solution set} = \{x : -1 \leq x < 5, x \in \mathbb{R}\}$$



$$\text{Solution set} = \{x : -1 \leq x < 5, x \in \mathbb{R}\}$$

**Ans.**

(c)

| Class Interval | Mid Value (x) | Frequency (f)   | fx                 |
|----------------|---------------|-----------------|--------------------|
| 20—30          | 25            | 10              | 250                |
| 30—40          | 35            | 6               | 210                |
| 40—50          | 45            | 8               | 360                |
| 50—60          | 55            | 12              | 660                |
| 60—70          | 65            | 5               | 325                |
| 70—80          | 75            | 9               | 675                |
|                |               | $\Sigma f = 50$ | $\Sigma fx = 2480$ |

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f} = \frac{2,480}{50} = 49.6$$

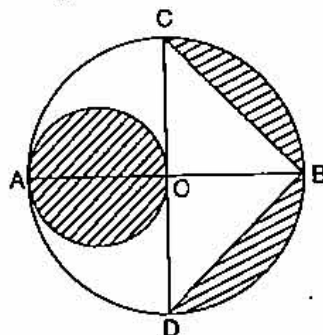
**Ans.**

### SECTION—B [40 Marks]

(Answer any four questions)

**Question 5.**

(a) In the given figure, AB is the diameter of a circle with centre O and  $OA = 7$  cm. Find the area of the shaded region. [3]



(b) Prove that  $\frac{\sin \theta \tan \theta}{1 - \cos \theta} = 1 + \sec \theta$ .

[3]

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(c) Mr. Ashok has an account in the Central Bank of India. The following entries are from his pass book :

| Date     | Particulars | Withdrawals |    | Deposits |    | Balance |    |
|----------|-------------|-------------|----|----------|----|---------|----|
|          |             | ₹           | P  | ₹        | P  | ₹       | P  |
| 01-01-05 | B/F         | —           | —  | —        | —  | 1200    | 00 |
| 07-01-05 | By cash     | —           | —  | 500      | 00 | 1700    | 00 |
| 17-01-05 | To cheque   | 400         | 00 | —        | —  | 1300    | 00 |
| 10-02-05 | By cash     | —           | —  | 800      | 00 | 2100    | 00 |
| 25-02-05 | To cheque   | 500         | 00 | —        | —  | 1600    | 00 |
| 20-09-05 | By cash     | —           | —  | 700      | 00 | 2300    | 00 |
| 21-11-05 | To cheque   | 600         | 00 | —        | —  | 1700    | 00 |
| 05-12-05 | By cash     | —           | —  | 300      | 00 | 2000    | 00 |

If Mr. Ashok gets ₹ 83.75 as interest at the end of the year where the interest is compounded annually, calculate the rate of interest paid by the bank in his Savings Bank Account on 31st December, 2005. [4]

**Solution.**

(a) Given : OA = 7 cm

Required Area = Area of small circle + Two segments

Area of small shaded circle =  $\pi r^2$ 

$$= \frac{22}{7} \times \frac{7 \times 7}{2 \times 2} = \frac{77}{2}$$

$$\left[ \text{where } r = \frac{7}{2} \text{ cm} \right]$$

$$= 38.5 \text{ cm}^2$$

Area of two shaded segments = Area of big semi-circle - Area of  $\Delta$  CBD

$$= \frac{\pi R^2}{2} - \frac{1}{2} \times b \times h$$

where R = 7 cm, b = CD = 14 cm, h = OB = 7 cm

$$= \frac{22}{7} \times \frac{7 \times 7}{2} - \frac{1}{2} \times 14 \times 7 = 77 - 49$$

$$= 28 \text{ cm}^2$$

Now

Required Area = 38.5 + 28

$$= 66.5 \text{ cm}^2.$$

**Ans.**

(b) Given :

L.H.S. =  $\frac{\sin \theta \tan \theta}{1 - \cos \theta}$

$$= \frac{\sin \theta \cdot \frac{\sin \theta}{\cos \theta}}{1 - \cos \theta}$$

$$= \frac{\sin^2 \theta}{1 - \cos \theta}$$

$$= \frac{\cos \theta (1 - \cos \theta)}{1 - \cos \theta}$$

$$= \frac{1 - \cos^2 \theta}{\cos \theta (1 - \cos \theta)}$$

$$= \frac{(1 - \cos \theta)(1 + \cos \theta)}{\cos \theta (1 - \cos \theta)}$$

$$= \frac{1 + \cos \theta}{\cos \theta}$$

$$= 1 + \sec \theta = \text{R.H.S.}$$

**Hence Proved.**

(c)

| Month        | Minimum Qualifying Balance between 10 <sup>th</sup> day and last day (in ₹) |
|--------------|---|
| January      | 1,300   |
| February     | 1,600   |
| March        | 1,600   |
| April        | 1,600   |
| May          | 1,600   |
| June         | 1,600   |
| July         | 1,600   |
| August       | 1,600   |
| September    | 1,600   |
| October      | 2,300   |
| November     | 1,700   |
| December     | 2,000   |
| <b>Total</b> | <b>₹ 20,100</b>   |

Now, Principal = ₹ 20,100, Time = 1 month =  $\frac{1}{12}$  years, Interest = ₹ 83.75

$$I = \frac{PRT}{100}$$

$$R = \frac{I \times 100}{P \times T} = \frac{83.75 \times 100 \times 12}{20100} = 5$$

∴ Rate of interest = 5%

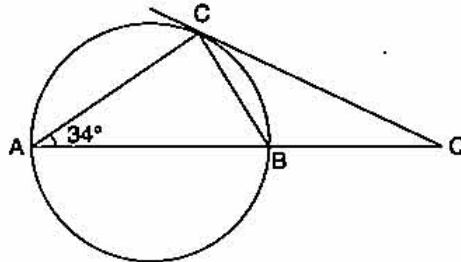
Ans.

### Question 6.

(a) In the given figure, AB is a diameter. The tangent at C meets AB produced at Q. If  $\angle CAB = 34^\circ$ , find :

(i)  $\angle CBA$ , (ii)  $\angle CQA$ .

[3]



(b) If the lines  $y = 3x + 7$  and  $2y + px = 3$  are perpendicular to each other, find the value of  $p$ . [3]

(c) Let  $A = \begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix}$ ,  $B = \begin{bmatrix} 0 & 2 \\ 1 & -1 \end{bmatrix}$  and  $C = \begin{bmatrix} -2 & 3 \\ 1 & -1 \end{bmatrix}$ . Find  $A^2 - A + BC$ . [4]

### Solution 6.

(a) (i) ∵ AB is diameter.

$$\therefore \angle ACB = 90^\circ \quad (\angle \text{ in a semicircle} = 90^\circ)$$

∴ In  $\Delta ABC$ ,

$$\begin{aligned} \therefore \angle CBA &= 180^\circ - (\angle BAC + \angle ACB) \\ &= 180^\circ - (34^\circ + 90^\circ) \quad (\text{sum of } \angle \text{s of a } \Delta = 180^\circ) \\ &= 56^\circ \end{aligned}$$

Ans.



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(ii)  $\therefore$  CQ is tangent.

$$\therefore \angle BCQ = \angle BAC = 34^\circ \quad (\text{Alternate segment angle property})$$

$$\angle CBQ = 180^\circ - \angle CBA$$

$$= 180^\circ - 56^\circ$$

$$= 124^\circ$$

In  $\Delta CBQ$ ,

$$\angle CQA = 180^\circ - (\angle BCQ + \angle CBQ)$$

$$\therefore = 180^\circ - (34^\circ + 124^\circ)$$

$$= 180^\circ - 158^\circ = 22^\circ$$

$$\therefore \angle CQA = 22^\circ.$$

Ans.

(b) Given :

$$y = 3x + 7$$

...(i)

$$\text{Slope of line } (m_1) = 3$$

$$2y + px = 3$$

...(ii)

$$2y = -px + 3$$

$$y = -\frac{p}{2}x + \frac{3}{2}$$

$$\text{Slope of line } (m_2) = \frac{-p}{2}$$

 $\therefore$  lines are perpendicular to each other.

$$\therefore m_1 \cdot m_2 = -1$$

$$3 \times -\frac{p}{2} = -1$$

$$p = \frac{2}{3}$$

Ans.

(c)

$$A^2 - A + BC = \begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix} \begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix} - \begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} -2 & 3 \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 16 - 12 & -8 + 6 \\ 24 - 18 & -12 + 9 \end{bmatrix} - \begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix} + \begin{bmatrix} 0 + 2 & 0 - 2 \\ -2 - 1 & 3 + 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix} - \begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix} + \begin{bmatrix} 2 & -2 \\ -3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 2 & -2 \\ -3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -2 \\ -3 & 4 \end{bmatrix}$$

Ans.

**Question 7.**

(a) Mr. Sanjeev Chopra gets a monthly salary of ₹ 16,000.\*\*

Savings : ● Contribution towards Provident Fund : ₹ 45,000 per year.

Donations : ● To Prime Minister's Relief Fund : ₹ 3,100 (eligible for 100% tax exemption)

\*\* Solution has not given due to out of present syllabus.

**Calculate :**

- (i) Mr. Chopra's taxable income.  
 (ii) The tax rebate on Mr. Chopra's savings.  
 (iii) The tax Mr. Chopra has to pay in the last month of the year, if he has been paying a tax of ₹ 1,200 per month in the first eleven months of the year.

**Tax slab :**

|                        |  |
|------------------------|--|
| Upto ₹ 50,000          | : No tax.  |
| ₹ 50,001 to ₹ 60,000   | : 10% of the income exceeding ₹ 50,000   |
| ₹ 60,001 to ₹ 1,50,000 | : ₹ 1,000 + 20% of the income exceeding ₹ 60,000                                   |
| Above ₹ 1,50,000       | : ₹ 19,000 + 30% of the income exceeding ₹ 1,50,000.                               |
| Standard Deduction     | : ₹ 20,000.  |
| Rebate in tax          | : 20% of the total savings or ₹ 14,000, whichever is less.                         |
| Surcharge              | : 10% of the total tax payable after rebate <span style="float: right;">[6]</span> |

- (b) The shadow of a vertical tower on a level ground increases by 10 m when the altitude of the sun changes from  $45^\circ$  to  $30^\circ$ . Find the height of the tower, correct to two decimal places. [4]

**Solution.**

- (b) Let height of tower AB be  $h$  and length of shadow initially be  $x$ .

$$\text{Now in } \triangle ACB, \tan 45^\circ = \frac{h}{x}$$

$$\therefore 1 = \frac{h}{x}$$

$$x = h$$

$$\text{In } \triangle ADB, \tan 30^\circ = \frac{h}{x + 10}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{h + 10}$$

$$h + 10 = \sqrt{3} h$$

$$\sqrt{3} h - h = 10$$

$$h(\sqrt{3} - 1) = 10$$

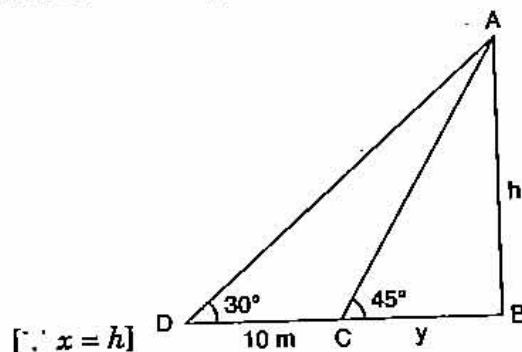
$$h = \frac{10(\sqrt{3} + 1)}{(\sqrt{3} - 1)(\sqrt{3} + 1)}$$

$$h = \frac{10(\sqrt{3} + 1)}{3 - 1}$$

$$h = \frac{10(1.732 + 1)}{2}$$

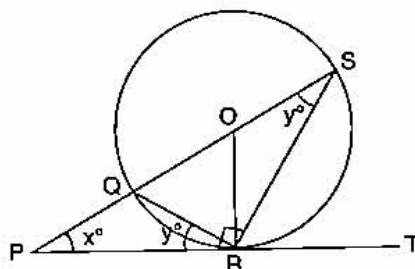
$$h = 5 \times 2.732$$

$$= 13.66 \text{ m.}$$

**Ans.**

**Question 8.**

- (a) In the given figure,  $PT$  touches a circle with centre  $O$  at  $R$ . Diameter  $SQ$  when produced meets  $PT$  at  $P$ . If  $\angle SPR = x^\circ$  and  $\angle QRP = y^\circ$ , show that  $x^\circ + 2y^\circ = 90^\circ$ . [3]



- (b) The line segment joining  $A(2, 3)$  and  $B(6, -5)$  is intersected by the  $X$ -axis at the point  $K$ . Write the ordinate of the point  $K$ . Hence find the ratio in which  $K$  divides  $AB$ . [3]
- (c) Mr. Ram Gopal invested ₹ 8,000 in 7% ₹ 100 shares at ₹ 80. After a year he sold these shares at ₹ 75 each and invested the proceeds (including his dividend) in 18%, ₹ 25 shares at ₹ 41. Find : [4]
- his dividend for the first year
  - his annual income in the second year
  - the percentage increase in his return on his original investment.

**Solution.**

- (a)  $PRT$  is tangent at  $R$ .  $QR$  is chord.

$$\therefore \begin{aligned} \angle QRP &= \angle RSQ = y^\circ \quad (\text{Alternate segment property}) \\ \angle QRS &= 90^\circ \\ &(\angle \text{ in a semi circle} = 90^\circ \text{ with diameter } QS) \end{aligned}$$

In  $\Delta PRS$ ,

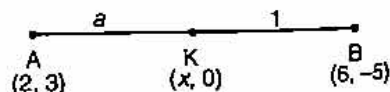
$$\begin{aligned} \angle SPR + \angle PRS + \angle RSP &= 180^\circ \\ x^\circ + (y^\circ + 90^\circ) + y^\circ &= 180^\circ \\ x^\circ + 2y^\circ &= 180^\circ - 90^\circ \\ x^\circ + 2y^\circ &= 90^\circ \end{aligned}$$

Hence Proved.

- (b) At  $X$ -axis,  $y = 0$

$\therefore$  Coordinate of point  $K = (x, 0)$

Let the point  $K$  divide  $AB$  in  $a : 1$



$$\begin{aligned} y &= \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} \\ 0 &= \frac{a \times -5 + 1 \times 3}{a + 1} \end{aligned}$$

$$5a = 3$$

$$a = \frac{3}{5}$$

$\therefore$   $K$  divides  $AB$  in the ratio =  $3 : 5$ .

Ans.

- (c) (i) Given : Investment = ₹ 8,000, MV of 1 share = ₹ 80

$$\therefore \text{No. of shares} = \frac{\text{Investment}}{\text{MV of 1 share}}$$

- $$= \frac{8,000}{80} = 100 \text{ shares}$$
- NV of 1 share = ₹ 100
- ∴ Total NV =  $100 \times 100 = ₹ 10,000$
- Dividend =  $\frac{7}{100} \times 10,000 = ₹ 700$
- ∴ Dividend for 1st year = ₹ 700. Ans.
- (ii) SP of 1 share = ₹ 75
- ∴ SP of all shares =  $75 \times 100 = ₹ 7,500$
- Total amount Investment =  $7,500 + 700 = ₹ 8,200$
- MV = ₹ 41
- NV = ₹ 25
- Rate of dividend = 18%
- No. of shares =  $\frac{\text{Investment}}{\text{MV}}$
- $$= \frac{8,200}{41} = 200$$
- Dividend =  $\frac{200 \times 25 \times 18}{100}$
- $$= ₹ 900$$
- ∴ Annual income in 2nd year = ₹ 900. Ans.
- (iii) Increase =  $900 - 700 = ₹ 200$
- ∴ Increase in percentage =  $\frac{\text{increase}}{\text{investment}} \times 100$
- $$= \frac{200}{8,000} \times 100$$
- $$= 2.5\%$$
- Ans.

**Question 9.**

- (a)  $A = \{-2, -1, 1, 2\}$  and  $f = \{(x, 1/x), x \in A\}^{**}$
- (i) List the domain of  $f$
- (ii) List the range of  $f$
- (iii) Is  $f$  a function? Give reasons for your answer. [3]
- (b) Solve the equation  $2x - \frac{1}{x} = 7$ . Write your answer correct to two decimal places. [3]
- (c) A vessel in the form of an inverted cone is filled with water to the brim. Its height is 20 cm and diameter is 16.8 cm. Two equal solid cones are dropped in it so that they are fully submerged. As a result, one third of the water in the original cone overflows. What is the volume of each of the solid cones submerged? [4]

**Solution.**

- (b) Given :
- $$2x - \frac{1}{x} = 7$$
- $$2x^2 - 1 = 7x$$
- $$2x^2 - 7x - 1 = 0$$

\*\* Solution has not given due to out of present syllabus.

Comparing with quadratic equation  $ax^2 + bx + c = 0$ , we get  $a = 2$ ,  $b = -7$ ,  $c = -1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(-1)}}{2 \times 2}$$

$$= \frac{7 \pm \sqrt{49 + 8}}{4}$$

$$= \frac{7 \pm \sqrt{57}}{4}$$

$$= \frac{7 + \sqrt{57}}{4} \quad \text{or} \quad \frac{7 - \sqrt{57}}{4}$$

$$= \frac{7 + 7.550}{4} \quad \text{or} \quad \frac{7 - 7.550}{4}$$

$$= 3.64 \quad \text{or} \quad -0.14$$

$$x = 3.64 \quad \text{or} \quad -0.14. \quad \text{Ans.}$$

(c) Given : Height of vessel = 20 cm, Radius of vessel =  $\frac{16.8}{2} = 8.4$  cm

$$\text{Volume of water in cone vessel} = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times 8.4 \times 8.4 \times 20$$

$$= 1478.4 \text{ cm}^3$$

$$\text{Volume of water overflown} = \frac{1}{3} \text{rd of original volume}$$

$$= \frac{1}{3} \times 1478.4$$

$$= 492.8 \text{ cm}^3$$

$$\text{Volume of water overflown} = \text{Volume of two equal solid cones}$$

$$492.8 = 2x$$

$$x = \frac{492.8}{2} = 246.4 \text{ cm}^3$$

$$\therefore \text{Volume of each solid cone} = 246.4 \text{ cm}^3. \quad \text{Ans.}$$

### Question 10.

(a) The daily wages of 160 workers in a building project are given below :

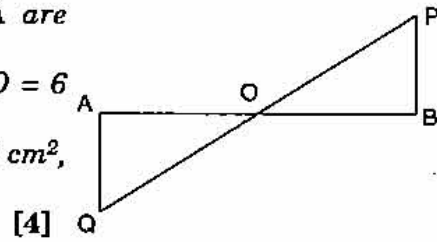
| Wages in ₹     | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-70 | 70-80 |
|----------------|------|-------|-------|-------|-------|-------|-------|-------|
| No. of Workers | 12   | 20    | 30    | 38    | 24    | 16    | 12    | 8     |

Using a graph paper, draw an Ogive for the above distribution.

Use your Ogive to estimate :

- the median wage of the workers.
- the upper quartile wage of the workers
- the lower quartile wages of the workers
- the percentage of workers who earn more than ₹ 45 a day.

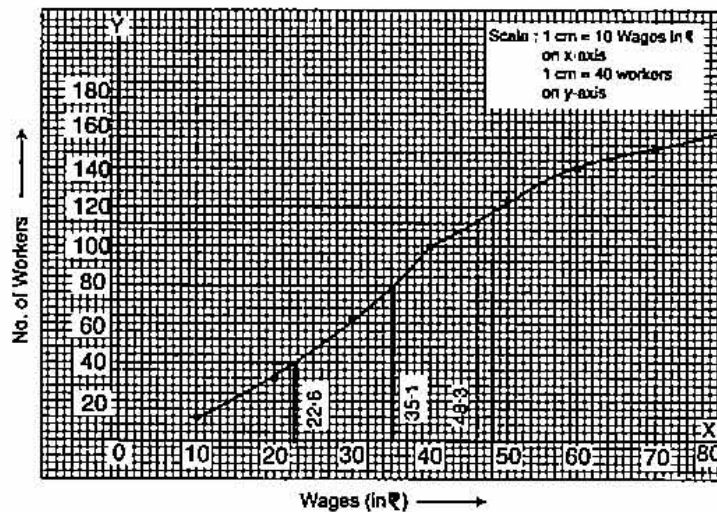
- (b) In the figure given alongside,  $PB$  and  $QA$  are perpendiculars to the line segment  $AB$ . If  $PO = 6$  cm,  $QO = 9$  cm and the area of  $\Delta POB = 120$  cm<sup>2</sup>, find the area of  $\Delta QOA$ .



[4]

**Solution.**

| (a) | Wages (in ₹) | No. of Workers | c.f. |
|-----|--------------|----------------|------|
|     | 0-10         | 12             | 12   |
|     | 10-20        | 20             | 32   |
|     | 20-30        | 30             | 62   |
|     | 30-40        | 38             | 100  |
|     | 40-50        | 24             | 124  |
|     | 50-60        | 16             | 140  |
|     | 60-70        | 12             | 152  |
|     | 70-80        | 8              | 160  |



(i) Median wage of the workers =  $\frac{N}{2} = \frac{160}{2} = 80$

Median = 35.1 or 35

Ans.

(ii) Upper quartile wage of the worker  $Q_3 = \frac{3N}{4} = 120$

Upper quartile = 48.3 or 48

Ans.

(iii) Lower quartile wage of the worker  $Q_1 = \frac{N}{4} = \frac{160}{4} = 40$

lower quartile = 22.6 or 23.

Ans.

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(iv) The percentage of workers earn more than ₹ 45

$$= \frac{160 - 112}{160} \times 100$$

$$= \frac{48 \times 10}{16} = \frac{480}{16}$$

$$= 30\%.$$

Ans.

(b) Area of  $\Delta POB = 120 \text{ cm}^2$ In  $\Delta AOQ$  and  $\Delta POB$ ,

$$\angle QAO = \angle OBP = 90^\circ$$

(Given : QA and PB are perpendicular to AB.)

$$\angle AOQ = \angle BOP \quad (\text{Vertically opposite } \angle s)$$

$$\Delta AOQ \sim \Delta POB \quad (\text{AA})$$

 $\therefore$ 

$$\Delta AOQ \sim \Delta POB$$

Now  $\therefore$  $\therefore$  By applying area theorem

$$\frac{\text{Area of } \Delta AOQ}{\text{Area of } \Delta POB} = \left( \frac{\text{side } OQ}{\text{side } PO} \right)^2$$

$$\therefore \text{Area of } \Delta AOQ = \left( \frac{9}{6} \right)^2 \times 120 = \frac{81 \times 120}{36}$$

$$= 270 \text{ cm}^2.$$

Ans.

**Question 11.**

(a) Find the coordinates of the centroid of a triangle whose vertices are :

$$A(-1, 3), B(1, -1) \text{ and } C(5, 1)$$

[3]

(b) Use a ruler and a pair of compasses to construct  $\Delta ABC$  in which  $BC = 4.2 \text{ cm}$ ,  $\angle ABC = 60^\circ$  and  $AB = 5 \text{ cm}$ . Construct a circle of radius 2 cm to touch both the arms of  $\angle ABC$  of  $\Delta ABC$ .

[3]

(c) A shopkeeper buys a certain number of books for ₹ 720. If the cost per book was ₹ 5 less, the number of books that could be bought for ₹ 720 would be 2 more. Taking the original cost of each book to be ₹  $x$ , write an equation in  $x$  and solve it.

[4]

**Solution.**

(a) Given : A(-1, 3), B(1, -1), C(5, 1)

Let coordinates of centroid be  $(x, y)$ .

$$\therefore (x, y) = \left( \frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$

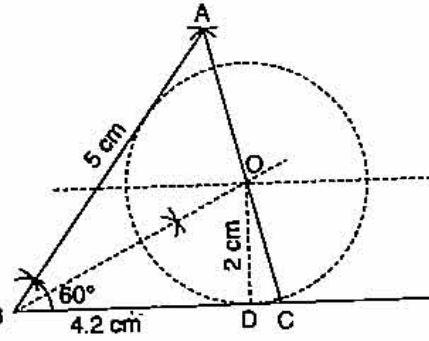
$$= \left( \frac{-1 + 1 + 5}{3}, \frac{3 - 1 + 1}{3} \right)$$

$$= \left( \frac{5}{3}, 1 \right)$$

Ans.

## (b) Steps of Constructions :

1. Draw BC of length 4.2 cm.
2. Draw an angle of  $60^\circ$  at B.
3. Cut BA = 4 cm and join A to C.
4. Draw an angle bisector of  $\angle ABC$ .
5. Draw a line  $\parallel$  to BC at 2 cm distance.
6. Angle bisector and parallel line intersect at O.
7. Taking O as centre and 2 cm as radius draw the required circle.



(c) Let original cost of each book = ₹  $x$

$$\therefore \text{No. of books for ₹ 720} = \frac{720}{x}$$

When cost decreases by 5 (i.e.,  $x - 5$ ) then

$$\text{No. of books} = \frac{720}{x-5}$$

$\therefore$  According to question,

$$\left( \frac{720}{x-5} \right) - \left( \frac{720}{x} \right) = 2$$

$$\frac{720x - 720(x-5)}{x(x-5)} = 2$$

$$720x - 720x + 3600 = 2(x^2 - 5x)$$

$$2x^2 - 10x - 3600 = 0$$

$$x^2 - 5x - 1800 = 0$$

$$x^2 + 40x - 45x - 1800 = 0$$

$$x(x+40) - 45(x+40) = 0$$

$$(x-45)(x+40) = 0$$

$$x = 45 \text{ and } -40$$

$x = -40$  is not possible.

$$\therefore \text{Original cost of each book} = ₹ 45.$$

Ans.



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