(An English medium plus 2 school of The Tagore Society, Jamshedpur)

MATHEMATICS

HALF-YEARLY-2015

F.M. 80 Time—2 1/2 hrs

(Attempt all questions from SECTION-A and any four questions from SECTION-B. Omission of essential working will result in loss of marks)

SECTION – A (4×10=40 marks)

Q1. (a) If $A = \begin{bmatrix} 4 & 4 \\ -2 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 3 & -2 \end{bmatrix}$, find the matrix $D$ such that $3A - 2B + 2D = 0$.

(b) Solve the following quadratic equation for $x$ and give your answer correct to three significant figures:

$$2x^2 - 4x - 3 = 0$$

(c) On a certain sum of money, the difference between the compound interest for a year, payable half-yearly and the simple interest for a year is Rs. 16. Find the sum lent out, if the rate of interest in both the cases is 8%.

Q2. (a) Find the value of $a$ and $b$ if $(x - 1)$ and $(x - 2)$ are the factors of $x^3 - ax + b$.

(b) Without using trigonometric tables evaluate:

$$\frac{4 \sin 32^\circ}{\cos 58^\circ} + \frac{5 \tan 48^\circ}{\cot 42^\circ} - \frac{8 \sec 72^\circ}{\cosec 18^\circ}$$

(c) Give the line segment $AB$ joining the points $A (-4, 6)$ and $B (8, -3)$. Find (i) the ratio in which $AB$ is divided by the y-axis (ii) the co-ordinates of the point of intersection (iii) the length of $AB$.

Q3. (a) Solve and graph the solution set on the number line: $2 \leq \frac{x - 4}{2} + \frac{x}{3} \leq 3; x \in \mathbb{I}$.

(b) An integer is chosen at random from 1 to 50. Find the probability that the number is (i) divisible by 5 (ii) a perfect cube (iii) a prime number.

(c) Mr. Jacob has a two years recurring deposit account in State Bank of India and deposits Rs. 1500 per month. If he receives Rs. 37,875 at the time of maturity, find the rate of interest.

Q4. (a) Prove that

$$\frac{1}{\sec x - \tan x} + \frac{1}{\sec x + \tan x} = \frac{2}{\cos x}$$

(b) What least number must be subtracted from each of the numbers 9, 15, 23 and 41 so that they are in proportion?

(c) Using graph paper and 1 cm = 1 unit along both x-axis and y-axis (i) Plot the points $A (-4, 4)$ and $B (2, 2)$, (ii) Reflect $A$ and $B$ in the origin to get the images $A'$ and $B'$ respectively. (iii) Write down the co-ordinates of $A'$ and $B'$. (iv) Give the geometric name of the figure $ABA'B'$. (v) Draw and name its lines of symmetry.

SECTION – B (4×10=40 marks) (Attempt any four questions)

Q5. (a) Two dice are rolled simultaneously. Find the probability of getting (i) a doublet (ii) a sum of 8 (iii) an odd number on the two faces of the dice.

(b) If $[\begin{array}{cc} 2 & 4 \\ 6 & 2 \end{array}] [\begin{array}{c} 3x \\ 2 \end{array}] + 2 [\begin{array}{c} 3 \\ 4 \end{array}] = 5 [\begin{array}{c} 4 \\ y \end{array}]$ find the values of $x$ and $y$.

(c) Using properties of proportion, solve for $x$: $\frac{\sqrt{2} + \sqrt{2} - x}{\sqrt{2} - \sqrt{2} - x} = 3$.

Q6. (a) If $A (4, 2)$, $B (6, 8)$ and $C (8, 4)$ are the vertices of $\triangle ABC$. Write down the equation of the median of the triangle through $A$.

(b) The printed price of an article is Rs. 60000. The wholesaler allows a discount of 20% to the shopkeeper. The shopkeeper sells the article to the customer at the printed price. Sales tax (under VAT) is charged at 8%.

(i) Find the VAT paid by the shopkeeper.

(ii) Find the amount paid by the customer to the shopkeeper.

(iii) Find the profit or loss for shopkeeper.
the rate of 6% at every stage. Find (i) the cost to the shopkeeper inclusive of tax (ii) VAT paid by the shopkeeper to the government (iii) the cost to the customer inclusive of tax.

(c) The sides AD, BC of a trapezium ABCD are parallel and the diagonals AC, BD meet at O. The area of \( \Delta AOB = 3 \text{cm}^2 \) and the area of \( \Delta BDC = 8 \text{cm}^2 \). Calculate the value of (i) area of \( \Delta BOC \) (ii) the ratio \( AO:OC \) (iii) area of \( \Delta AOD \).

\[ (3+3+4) \]

Q 7. (a) Two pillars are of equal height and on either sides of the road, which is 100m wide. The angles of elevation of the top of the pillars are 60\(^\circ\) and 30\(^\circ\) at a point on the road between the pillars. Find the position of the point between the pillars and the height of each pillar.

(b) A page of the pass book of Ravi, who has a savings bank account in PNB is given below:

<table>
<thead>
<tr>
<th>Date</th>
<th>Particulars</th>
<th>Withdrawals (Rs.)</th>
<th>Deposits (Rs.)</th>
<th>Balance (Rs.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.01.2008</td>
<td>By cash</td>
<td>-</td>
<td>2400</td>
<td>2400</td>
</tr>
<tr>
<td>9.01.2008</td>
<td>By cash</td>
<td>-</td>
<td>1600</td>
<td>4000</td>
</tr>
<tr>
<td>8.02.2008</td>
<td>To self</td>
<td>400</td>
<td>-</td>
<td>5400</td>
</tr>
<tr>
<td>21.04.2008</td>
<td>To cheque</td>
<td>1700</td>
<td>-</td>
<td>7100</td>
</tr>
<tr>
<td>1.05.2008</td>
<td>By clearing</td>
<td>-</td>
<td>3000</td>
<td>9100</td>
</tr>
<tr>
<td>17.05.2008</td>
<td>By cash</td>
<td>-</td>
<td>2000</td>
<td>9100</td>
</tr>
</tbody>
</table>

Complete the balance. The interest received at the end of the June 2008 is Rs. 124.50. Find the rate of interest per annum.

\[ (5+5) \]

Q 8 (a) Solve the following inequation and represent the solution set on the number line:

\[ 4x - 19 < \frac{3x}{5} - 2, \leq \frac{-2}{5} + x, x \in R \]

(b) Without solving the following quadratic equation, find the value of 'm' for which the given equation has real and equal roots. \( x^2 + 2(m - 1)x + (m + 5) = 0 \)

(c) The model of a building is constructed with scale factor 1.30. (i) If the height of the model is 80 cm, find the actual height of the building in metres. (ii) If the actual volume of a tank at the top of the building is \( \leq 7 \) m\(^3\), find the volume of the tank on the top of the model.

\[ (3+3+4) \]

Q 9 (a) If \( A = \begin{bmatrix} 2 & -1 \\ -4 & 5 \end{bmatrix} \) and \( B = \begin{bmatrix} -3 \\ 2 \end{bmatrix} \), find matrix \( C \) such that \( AC = B \).

(b) If the rate of sales tax is 5%, Simran has to pay Rs. 7,140 for the steel cupboard. What amount she has to pay if the sales tax is increased by 2%?

(c) Find the co-ordinates of the foot of the perpendicular drawn from the point (1, -2) on the line \( y = 2x + 1 \).

\[ (3+3+4) \]

Q 10 (a) Find the probability that the month of January may have 5 M\( \text{o}^{\text{os}} \) (i) in a leap year (ii) a non-leap year.

(b) Construct a regular hexagon of side 3 cm, draw all the lines of symmetry.

(c) If \( (x - 2) \) is a factor of the expression \( 2x^3 + ax^2 + bx - 14 \) and when the expression is divided by \( (x - 3) \), it leaves a remainder 52, Find the value of \( a \) and \( b \).

\[ (3+3+4) \]

Q 11 (a) Prove \[ \frac{\cos \theta - \sin \theta (1 + \tan \theta)}{2 \cos^2 \theta - 1} = \sec \theta \]

(b) Find the equation of the line perpendicular to \( 5x - y \) and which passes through the midpoint of the line segment joining (2, 3) and (4, 5).

(c) The difference between the compound interest and the simple interest on Rs. 42000 for two years is Rs. 105 at the same rate of interest per annum, find (i) the rate of interest (ii) the compound interest earned in the second year.

\[ (3+3+4) \]