CBSE 12th Mathematics 2014 Unsolved Paper

Delhi Board

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TIME - 3HR. | QUESTIONS - 29

THE MARKS ARE MENTIONED ON EACH QUESTION

SECTION – A

Question number 1 to 10 carry 1 marks each.

Q.1. Let * be a binary operation, on the set of all non- zero real number, given by

$$a * b = \frac{ab}{5} \; for\; a, b \in R - \{0\}.$$

Find the value of x,given that $2 * (x * 5) = 10$. *1 marks*

Q. 2. If $\sin\left(sin^{-1}\frac{1}{5} + cos^{-1}x\right) = 1$, then find the value of x. *1 marks*

Q. 3. If $2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$, $find\ (x - y)$. *1 marks*

Q. 4. Solve the following matrix equation *1 marks*

$$\text{for } x:, \quad [x \quad 1].\begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix} = 0.$$

Q. 5. $If \begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$, $write\ the\ value\ of\ x$. *1 marks*

Q. 6. Write the antiderivative of $\left(3\sqrt{x} + \frac{1}{\sqrt{x}}\right)$. *1 marks*

Q. 7. Evaluate: *1 marks*

$$\int_0^3 \frac{dx}{9 + x^2}.$$

Q. 8. Find the projection of the vector $\hat{\imath} + 3\hat{\jmath} + 7\hat{k}$ on the vector

$2\hat{\imath} - 3\hat{\jmath} + 6\hat{k}$. *1marks*

Q. 9. $If\ \vec{a}\ and\ \vec{b}$ are two unit vectors such that $\vec{a} + \vec{b}$ is also a unit vector, then find the angle between $\vec{a}\ and\ \vec{b}$. *1 marks*

Q.10. write the vector equation of the plane, passing through the point (a, b, c) and Parallel to the plane $\vec{r}\,.\left(\hat{\imath} + \hat{\jmath} + \hat{k}\right) = 2.$ *1 marks*

SECTION-B

Question number 11 to 22 carry 4 marks each:

Q. 11. $Let\ A = \{1, 2, 3, \ldots\ldots, \}$ **and R be the relation in A × A defined by** $(a, b)\ R\ (c, d)$ **if** $a + d = b + c$ **for** $(a, b), (c, d)\ in$ **A ×A. Prove that R is an equivalence relation. Also obtain the equivalence class [(2,5)].** *4 marks*

Q. 12. Prove that *4 marks*

$$cot^{-1}\left(\frac{\sqrt{1 + sin\,x} + \sqrt{1 - sin\,x}}{\sqrt{1 + sin\,x} - \sqrt{1 - sin\,x}}\right) = \frac{x}{2}; x\,\epsilon\,\left(0, \frac{\pi}{4}\right).$$

OR

Prove that

$$2\,tan^{-1}\left(\frac{1}{5}\right) + sec^{-1}\left(\frac{5\sqrt{2}}{7}\right) + 2\,tan^{-1}\left(\frac{1}{8}\right) = \frac{\pi}{4}.$$

Q. 13. Using properties of determinants, prove that *4 marks*

$$\begin{vmatrix} 2y & y - z - x & 2y \\ 2z & 2z & z - x - y \\ x - y - z & 2x & 2x \end{vmatrix} = (x + y + z)^3$$

Q.14. Differentiate $tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$ **with respect to** $\cos^{-1}\left(2x\sqrt{1 - x^2}\right)$, **when** $x \neq 0.$ *4 marks*

Q.15. If $y = x^x$, **prove that** *4 marks*

$$\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0.$$

Q.14. Find the intervals in which the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 5\ is$ *4 marks*

(a) strictly increasing

(b) strictly decreasing

OR

Find the equations of the tangent and normal to the curve $x = a\,sin^3\,\theta\ and\ y = a\,cos^3\theta\ at\ \theta = \frac{\pi}{4}$.

Q17. Evaluate: 4 marks

$$\int \frac{\sin^6 x \cos^6 x}{\sin^2 x \; \cos^2 x}\, dx.$$

OR

Evaluate: $\int (x-3)\sqrt{x^2+3x-18}\, dx.$

Q.18. Find the particular solution of the differential equation $e^x\sqrt{1-y^2}\, dx + \frac{y}{x}\, dy = 0$, **given that** $y = 1$ **when** $x = 0$, 4 marks

Q.19. Solve the following differential equation: 4 marks

$$(x^2-1)\frac{dy}{dx} + 2xy = \frac{2}{x^2-1}.$$

Q.20. Prove that, for any three vectors $\vec{a}, \vec{b}, \vec{c}$. 4 marks

$$\left[\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}\right] = 2\,[\vec{a}, \vec{b}, \vec{c}]$$

OR

Vectors $\vec{a}, \vec{b}$ **and** $\vec{c}$ **are such that** $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ **and** $|\vec{a}| = 3, |\vec{b}| = 5$ **and** $|\vec{c}| = 7$ **find the angle between** $\vec{a}$ **and** $\vec{b}$.

Q.21. Show that the lines: 4 marks

$$\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7} \text{ and } \frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5}$$

Intersect. Also, find their point of intersection.

Q.22. Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls? Given that 4 marks

(i) the youngest is a girl.

(ii) at least one is a girl.

SECTION – C

Question number 11 to 22 carry 4 marks each:

Q.23. Two school's P and Q want to award their selected students on the values of discipline, politeness and punctuality. The school P wants to award Rs y each and Rs z each for the three respective values to its 3, 2 and 1 students with total award money of Rs 1,000. School Q wants to spend Rs 1,500 to award its 4, 1 and 3 students on the respective values (by giving the same award money for the three values as

before). If the total amount of awards for one prize on each value is Rs 600, using matrices, find the award money for each value.

A part from the above three values, suggest one more value for awards. *6 marks*

Q.24. Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is $cos^{-1}\frac{1}{\sqrt{3}}$. *6 marks*

Q. 25. Evaluate: *6 marks*

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{cotx}}.$$

Q. 26. Find the area of the region in the first quadrant enclosed by the $x - axis$,

The line $y = x$ and the circle $x^2 + y^2 = 32$. *6 marks*

Q.27. Find the distance between the point (7,2,4) and the plane determined by the points A (2,5-3), B (-2, -3,5) and C (5,3, -3). *6 marks*

OR

Find the distance of the point (-1, -5, -10) from the point of intersection of the line

$\vec{r} = 2\hat{\imath} - \hat{\jmath} + 2\hat{k} + \lambda(3\hat{\imath} + 4\hat{\jmath} + 2\hat{k})$ and the plane $\vec{r}.(\hat{\imath} - \hat{\jmath} + \hat{k}) = 5$.

Q.28. A dealer in rural area wishes to purchase a number of sewing machine. He has only Rs5,760 to invest and has space for at most 20 items for strong. An electronic sewing machine cost him Rs360 and a manually operated sewing machine Rs240. He can sell an electronic sewing machine at a profit of Rs22 and a manually operated sewing machine at a profit of Rs18. Assuming that he can sell all the items that he can buy, how should he invest his money in order to maximize his profit? Make it as a LPP and solve it graphically. *6 marks*

Q. 29. A card from a pack of 52 playing cards is lost. From the remaining cards of the pack, three cards are drawn at random (without replacement) and are found to be all spades. Find the probability of the lost card being a spade. *6 marks*

OR

From a lot of 15 bulbs which include 5 defectives, a sample of 4 bulbs is drawn one by one with replacement. Find the probability distribution of number of defective bulbs. Hence find the mean of the distribution.


All
the Best