CBSE 12th Mathematics 2017 Unsolved Paper Delhi Board

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TIME - 3HR. | QUESTIONS - 29

THE MARKS ARE MENTIONED ON EACH QUESTION

SECTION – A

Question numbers 1 to 4 carry 1 mark each.

Question 1:

If A is a 3 × 3 invertible matrix, then what will be the value of k if $\det(A^{-1}) = (\det A)^k$. *1mark*

Question 2:

Determine the value of the constant 'k' so that the function *1mark*

$$f(x) = \begin{cases} \frac{kx}{|x|}, & if\ x < 0 \\ 3, & if\ x \geq 0 \end{cases} \quad is\ continuous\ at\ x = 0$$

Question 3:

Evaluate: $\int_2^3 3^x dx$ *1mark*

Question 4:

If a line makes angles 90° *and* 60° **respectively with the positive directions of x and y axis, find the angle which it makes with the positive direction of z-axis.** *1mark*

SECTION - B

Question numbers 5 to 12 carry 2 mark each.

Question 5:

Show that all the diagonal elements of a skew symmetric matrix are zero. *2 marks*

Question 6:

Find $\frac{dy}{dx}$ $at\ x = 1, y = \frac{\pi}{4}\ if\ \sin^2 y + \cos xy = K$ *2 marks*

Question 7:

The volume of a sphere is increasing at the rate of 3 cubic centimeter per second. Find the rate of increase of its surface area, when the radius is 2cm. *2 marks*

Question 8:

Show that the function $f(x) = 4x^3 - 18x^2 + 27x - 7$ is always increasing on R. *2 marks*

Question 9:

Find the vector equation of the line passing through the point A(1, 2, -1) and parallel to the line $5x - 25 = 14 - 7y = 35z$. *2 marks*

Question 10:

Prove that if E and F are independent events, then the events E and F$'$ are also independent. *2 marks*

Question11:

A small firm manufactures necklaces and bracelets. The total number of necklaces and bracelets that it can handle per day is at most 24. It takes one hour to make a bracelet and half an hour to make a necklace. The maximum number of hours available per day is 16. If the profit on a necklace is ₹ 100 and that on a bracelet is ₹ 300. Formulate on L.P.P. for finding how many of each should be produced daily to maximize the profit? It is being given that at least one of each must be produced. *2 marks*

Question12:

Find: *2 marks*

$$\int \frac{dx}{x^2 + 4x + 8}$$

SECTION - C

Question numbers 13 to 23 carry 4 mark each.

Question13:

Prove that $\tan\left\{\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right\}+\tan\left\{\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right\}=\frac{2b}{a}$ *4 marks*

Question14:

Using properties of determinants, prove that *4 marks*

$$\begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix} = 9y^2(x+y)$$

OR

Let $A=\begin{pmatrix}2 & -1\\ 3 & 4\end{pmatrix}, B=\begin{pmatrix}5 & 2\\ 7 & 4\end{pmatrix}, C=\begin{pmatrix}2 & 5\\ 3 & 8\end{pmatrix}$, **Find a matrix D such that** $\mathrm{CD}-\mathrm{AB}=0$

Question15:

Differentiate the function $(\sin x)^x + \sin^{-1}\sqrt{x}$ **with respect to** x. *4 marks*

OR

If $x^m y^n = (x+y)^{m+n}, proe\ that\ \frac{d^2y}{dx^2} = 0.$

Question 16:

Find: *4 marks*

$$\int \frac{2x}{(x^2+1)(x^2+2)^2}dx$$

Question 17:

Evaluate: *4 marks*

$$\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}\,dx$$

OR

Evaluate:

$$\int_0^{3/2} |x \sin \pi x|\, dx$$

Question 18:

Prove that $x^2 - y^2 = c(x^2 + y^2)^2$ **is the general solution of the differential equation** $(x^3 - 3xy^2)dx = (y^3 - 3x^2y)dy$**, where C is a parameter.** *4 marks*

Question 19:

Let $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$, $\vec{b} = \hat{\imath}$ *and* $\vec{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$, **then**

(a) Let $c_1 = -1$ *and* $c_2 = 2$, *find* c_3 *which makes* $\vec{a}, \vec{b}$ *and* $\vec{c}$ *coplanar*.

(b) If $c_2 = -1$ *and* $c_3 = 1$, *4 marks*

show that no value of c_1 ***can make*** $\vec{a}, \vec{b}$ ***and*** $\vec{c}$ ***coplanar.***

Question 20:

$\vec{a}, \vec{b}, \vec{c}$ **are mutually perpendicular vectors of equal magnitudes show that the vector** $\vec{a} + \vec{b} + \vec{c}$ **is equally inclined to** $\vec{a}, \vec{b}$ *and* $\vec{c}$**. Also, find the angle which** $\vec{a} + \vec{b} + \vec{c}$ **makes with** $\vec{a}$ **or** $\vec{b}$ **or** $\vec{c}$**.** *4 marks*

Question 21:

The random variable X can take only the values 0, 1, 2, 3. Given that $P(X = 0) = P(X = 1) = p$ *and* $P(X = 2) = p(X = 3)$ **such that** $\sum p_i x_i^2 = 2 \sum p_i x_i$**, find the value of p.** *4 marks*

Question 22:

Often it is taken that a truthful person commands, more respect in the society. A man is known to speak the truth 4 out of 5 times. He throws a die and reports that it is a six.

Find the probability that it is actually a six. *4 marks*

Do you also agree that the value of truthfulness leads to more respect in the society?

Question 23:

Solve the above LPP graphically. *4 marks*

Minimize $Z = 5x + 10y$

Subject to $x + 2y \leq 120$

the constraints $x + y \geq 60$

$x - 2y \geq 0$

and $x, y \geq 0$

SECTION - D

Question numbers 24 to 29 carry 6 mark each.

Question 24:

Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ **to solve the system of equations** *6 marks*

$x + 3z = 9, -x + 2y - 2z = 4, \quad 2x - 3y + 4z = -3.$

Question 25:

Consider $f: \mathrm{R} \to [-5, \infty)$ *given by* $f(x) = 9x^2 + 6x - 5.$ *6 marks*

Show that f is invertible with $f^{-1}(\mathrm{y}) = \left(\sqrt{\frac{y+6-1}{3}}\right)$

Hence Find

(i) $f^{-1}(\mathrm{y})$

(ii) $y \text{ if } f^{-1}(\mathrm{y}) = \frac{4}{3}$,

Where R_+ **is the set of all non-negative real numbers.**

OR

Discuss the commutativity and associativity of binary operation '*' defined on $A = Q - \{1\}$ **by the rule** $a * b = a - b + ab$ **for all** $a, b \in$ **A. Also find the identity element of * in A and hence find the invertible elements of A.**

Question 26:

If the sum of lengths of the hypotenuse and a side of a right angled triangle is given show that the area of the triangle is maximum, when the angle between them is $\frac{\pi}{3}$
6 marks

Question 27:

Using integration, find the area of region bounded by the triangle whose vertices are (-2, 1), (0, 4) and (2, 3). *6 marks*

OR

Find the area bounded by the circle $x^2 + y^2 = 16$ and the line $\sqrt{3y} = x$ in the first quadrant, using integration.

Question 28:

Solve the differential equation $x\frac{dy}{dx} + y = x\cos x + \sin x$, given that $y = 1$ when $x = \frac{\pi}{2}$ *6 marks*

Question 29:

Find the equation of the plane through the line of intersection of

$\vec{r}.(2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}) = 1$ and $\vec{r}.(\hat{\imath} - \hat{\jmath}) + 4 = 0$ and perpendicular to the plane $\vec{r}.(2\hat{\imath} - \hat{\jmath} + \hat{k}) + 8 = 0$. Hence find whether the plane thus obtained contains the line $x - 1 = 2y - 4 = 3z - 12$. *6 marks*

OR

Find the vector and Cartesian equations of a line passing through (1, 2, -4) and perpendicular to the two lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ *and* $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$


All
the Best